11 General Relativistic Models for Propagation (1 June 2004)

11.1 VLBI Time Delay

11.1.1 Historical Background

To resolve differences between numerous procedures used in the 1980s to model the VLBI delay, and to arrive at a standard model, a workshop was held at the U. S. Naval Observatory on 12 October 1990. The proceedings of this workshop have been published (Eubanks, 1991) and the model given there was called the 'consensus model.' It was derived from a combination of five different relativistic models for the geodetic delay. These are the Masterfit/Modest model, due to Fanselow and Thomas (see Treuhaft and Thomas, in Eubanks (1991), and Sovers and Fanselow (1987)), the I. I. Shapiro model (see Ryan, in Eubanks, (1991)), the Hellings-Shahid-Saless model (Shahid-Saless *et al.*, 1991) and in Eubanks (1991), the Soffel, Muller, Wu and Xu model (Soffel *et al.*, 1991) and in Eubanks (1991), and the Zhu-Groten model (Zhu and Groten, 1988) and in Eubanks (1991). At the same epoch, a relativistic model of VLBI observations was also presented in Kopeikin (1990) and in Klioner (1991).

The 'consensus model' formed the basis of that proposed in the IERS Standards (McCarthy, 1992). Over the years, there was considerable discussion and misunderstanding on the interpretation of the stations' coordinates obtained from the VLBI analyses. Particularly the IERS Conventions (McCarthy, 1996) proposed a modification of the delay, erroneously intending to comply with the XXIIst General Assembly of the International Astronomical Union in 1991 and the XXIst General Assembly of the International Union of Geodesy and Geophysics in 1991 Resolutions defining the Geocentric reference system. It seems, however, that this modification was not implemented by IERS analysis centers.

In the presentation below, the model is developed in the frame of the IAU Resolutions *i.e.* general relativity ($\gamma = 1$) using the Barycentric Celestial Reference System (BCRS) and Geocentric Celestial Reference System (GCRS) (as defined in the Appendix). However two approaches are presented for its usage, depending on the choice of coordinate time in the geocentric system. It is discussed how the Terrestrial Reference System (TRS) VLBI station coordinates submitted to the IERS, and the resulting ITRF2000 coordinates (Chapter 4), should be interpreted in relation to the IAU and IUGG Resolutions.

The 'step-by-step' procedure presented here to compute the VLBI time delay is taken from (Eubanks, 1991) and the reader is urged to consult that publication for further details.

11.1.2 Specifications and Domain of Application

The model is designed primarily for the analysis of VLBI observations of extra-galactic objects acquired from the surface of the Earth.[1] All terms of order 10^{-13} seconds or larger are included to ensure that the final result is accurate at the picosecond level. It is assumed that a linear combination of dual frequency measurements is used to remove the dispersive effect of the ionosphere, so that atmospheric effects are only due to the troposphere.

The model is not intended for use with observations of sources in the solar system, nor is it intended for use with observations made from space-based VLBI, from either low or high Earth orbit, or from the surface of the Moon (although it would be suitable with obvious changes for observations made entirely from the Moon).

The geocentric celestial reference system (GCRS) is kinematically non-rotating (not dynamically non-rotating) and, included in the precession constant and nutation series, are the effects of the geodesic precession ($\sim$ 19 milli arc seconds / y). If needed, Soffel *et al.* (1991) and Shahid-Saless *et al.* (1991) give details of a dynamically inertial VLBI delay equation. At the picosecond level, there is no practical difference for VLBI geodesy and astrometry except for the adjustment in the precession constant.

[1] The case of radio sources inside our galaxy has been considered in *e.g.* Sovers and Fanselow (1987); Klioner (1991)

11.1.3 The Analysis of VLBI Measurements: Definitions and Interpretation of Results

In principle, the observable quantities in the VLBI technique are recorded signals measured in the proper time of the station clocks. On the other hand, the VLBI model is expressed in terms of coordinate quantities in a given reference system (see Chapter 10 for a presentation of the different coordinate times used). For practical considerations, particularly because the station clocks do not produce ideal proper time (they even are, in general, synchronized and syntonized to UTC to some level, *i.e.* they have the same rate as the coordinate time Terrestrial Time (TT)), the VLBI delay produced by a correlator center may be considered to be, within the uncertainty aimed at in this chapter, equal to the TT coordinate time interval d_{TT} between two events: the arrival of a radio signal from the source at the reference point of the first station and the arrival of the same signal at the reference point of the second station. Note that we model here only the propagation delay and do not account for the desynchronization or desyntonization of the station clocks. From a TT coordinate interval, d_{TT}, one may derive a Geocentric Coordinate Time (TCG) coordinate interval, d_{TCG}, by simple scaling: $d_{TCG} = d_{TT}/(1 - L_G)$, where L_G is given in Table 1.1. In the following, two different approaches are presented using two different geocentric coordinate system with either TCG or TT as coordinate time.

The VLBI model presented below (formula (9)) relates the TCG coordinate interval $d_{TCG} = t_{v_2} - t_{v_1}$ to a baseline $\vec{b}$ expressed in GCRS coordinates (see the definition of notations in the next section). In the first approach, therefore, if the VLBI delay was scaled to a TCG coordinate interval, as described above, the results of the VLBI analysis would be directly obtained in terms of the spatial coordinates of the GCRS, as is recommended by the IUGG Resolution 2 (1991) and IAU Resolution B6 (1997), *i.e.* one would obtain TRS coordinates that are termed "consistent with TCG," here denoted x_{TCG}.

In the second approach, if the VLBI model (formula (9)) is used with VLBI delays as directly provided by correlators (*i.e.* equivalent to a TT coordinate interval d_{TT} without transformation to TCG), the baseline $\vec{b}$ is not expressed in GCRS but in some other coordinate system. The transformation of these coordinates to GCRS reduces, at the level of uncertainty considered here, to a simple scaling. The TRS space coordinates resulting from the VLBI analysis (here denoted x_{VLBI}) are then termed "consistent with TT" and the TRS coordinates recommended by the IAU and IUGG resolutions, x_{TCG}, may be obtained *a posteriori* by $x_{TCG} = x_{VLBI}/(1 - L_G)$ (see Petit, 2000).

All VLBI analysis centers submitting to the IERS have used this second approach and, therefore, the VLBI space coordinates are of the type x_{VLBI}. For continuity, an ITRF workshop (November 2000) decided to continue to use this approach, making it the present conventional choice for submission to the IERS. Note that the use of space coordinates "consistent with TT" is also the present conventional choice of SLR analysis results submitted to the IERS. At the ITRF workshop, it was also decided that the coordinates should not be re-scaled to x_{TCG} for the computation of ITRF2000 (see Chapter 4) so that the scale of ITRF2000 **does not comply with IAU and IUGG resolutions**.

11.1.4 The VLBI Delay Model

Although the delay to be calculated is the time of arrival at station 2 minus the time of arrival at station 1, it is the time of arrival at station 1 that serves as the time reference for the measurement. Unless explicitly stated otherwise, all vector and scalar quantities are assumed to be calculated at t_1, the time of arrival at station 1 including the effects of the troposphere. The VLBI hardware provides the UTC time tag for this event. For quantities such as $\vec{X}_J$, $V_\oplus$, $\vec{w}_i$, or U it is assumed that a table (or numerical formula) is available as a function of a given time argument. The UTC time tag should be transformed to the appropriate timescale corresponding to the time argument to be used to compute each element of the geometric model.

The baseline vector $\vec{b}$ is given in the kinematically non-rotating GCRS. It must be transformed to the rotating terrestrial reference frame defined in Chapter 4 of the present VLBI Conventions in accordance to the transformations introduced in Chapter 5.

Table 11.1 Notation used in the model.

t_i	the TCG time of arrival of a radio signal at the i^{th} VLBI receiver
T_i	the TCB time of arrival of a radio signal at the i^{th} VLBI receiver
t_{g_i}	the "geometric" TCG time of arrival of a radio signal at the i^{th} VLBI receiver including the gravitational "bending" delay and the change in the geometric delay caused by the existence of the atmospheric propagation delay but neglecting the atmospheric propagation delay itself
t_{v_i}	the "vacuum" TCG time of arrival of a radio signal at the i^{th} VLBI receiver including the gravitational delay but neglecting the atmospheric propagation delay and the change in the geometric delay caused by the existence of the atmospheric propagation delay
δt_{atm_i}	the atmospheric propagation TCG delay for the i^{th} receiver $= t_i - t_{g_i}$
T_{i_J}	the approximation to the TCB time that the ray path to station i passed closest to gravitating body J
ΔT_{grav}	the differential TCB gravitational time delay
$\vec{x}_i(t_i)$	the GCRS radius vector of the i^{th} receiver at t_i
$\vec{b}$	$\vec{x}_2(t_1) - \vec{x}_1(t_1)$ and is thus the GCRS baseline vector at the time of arrival t_1
$\delta\vec{b}$	a variation (*e.g.* true value minus *a priori* value) in the GCRS baseline vector
$\vec{w}_i$	the geocentric velocity of the i^{th} receiver
$\hat{K}$	the unit vector from the barycenter to the source in the absence of gravitational or aberrational bending
$\hat{k}_i$	the unit vector from the i^{th} station to the source after aberration
$\vec{X}_i$	the barycentric radius vector of the i^{th} receiver
$\vec{X}_\oplus$	the barycentric radius vector of the geocenter
$\vec{X}_J$	the barycentric radius vector of the J^{th} gravitating body
$\vec{R}_{i_J}$	the vector from the J^{th} gravitating body to the i^{th} receiver
$\vec{R}_{\oplus_J}$	the vector from the J^{th} gravitating body to the geocenter
$\vec{R}_{\oplus\odot}$	the vector from the Sun to the geocenter
$\hat{N}_{i_J}$	the unit vector from the J^{th} gravitating body to the i^{th} receiver
$\vec{V}_\oplus$	the barycentric velocity of the geocenter
U	the gravitational potential at the geocenter, neglecting the effects of the Earth's mass. At the picosecond level, only the solar potential need be included in U so that $U = GM_\odot/\lvert\vec{R}_{\oplus\odot}\rvert$
M_i	the rest mass of the i^{th} gravitating body
$M_\oplus$	the rest mass of the Earth
c	the speed of light
G	the Gravitational Constant

Vector magnitudes are expressed by the absolute value sign $[\lvert x\rvert = (\Sigma x_i^2)^{\frac{1}{2}}]$. Vectors and scalars expressed in geocentric coordinates are denoted by lower case (*e.g.* $\vec{x}$ and t), while quantities in barycentric coordinates are in upper case (*e.g.* $\vec{X}$ and T). A lower case subscript (*e.g.* $\vec{x}_i$) denotes a particular VLBI receiver, while an upper case subscript (*e.g.* $\vec{x}_J$) denotes a particular gravitating body. The SI system of units is used throughout.

(a) Gravitational Delay[2]

The general relativistic delay, ΔT_{grav}, is given for the J^{th} gravitating body by

$$\Delta T_{grav_J} = 2\frac{GM_J}{c^3}\ln\frac{\lvert\vec{R}_{1_J}\rvert + \vec{K}\cdot\vec{R}_{1_J}}{\lvert\vec{R}_{2_J}\rvert + \vec{K}\cdot\vec{R}_{2_J}}. \tag{1}$$

At the picosecond level it is possible to simplify the delay due to the Earth, $\Delta T_{grav_\oplus}$, which

[2]The formulas in this section are unchanged from the previous edition of the Conventions. The more advanced theory in Kopeikin and Schäfer (1999) provides a rigorous physical solution for the light propagation in the field of moving bodies. For Earth-based VLBI, the formulas in this section and those proposed in Kopeikin and Schäfer (1999) are numerically equivalent with an uncertainty of 0.1 ps (Klioner and Soffel, 2001).

becomes

$$\Delta T_{grav_\oplus} = 2\frac{GM_\oplus}{c^3}\ln\frac{|\vec{x}_1| + \vec{K}\cdot\vec{x}_1}{|\vec{x}_2| + \vec{K}\cdot\vec{x}_2}. \tag{2}$$

The Sun, the Earth and Jupiter must be included, as well as the other planets in the solar system along with the Earth's Moon, for which the maximum delay change is several picoseconds. The major satellites of Jupiter, Saturn and Neptune should also be included if the ray path passes close to them. This is very unlikely in normal geodetic observing but may occur during planetary occultations. Note that in case of observations very close to some massive bodies, extra terms (*e.g.* due to the multipole moments and spin of the bodies) should be taken into account to obtain an uncertainty of 1 ps (see Klioner, 1991).

The effect on the bending delay of the motion of the gravitating body during the time of propagation along the ray path is small for the Sun but can be several hundred picoseconds for Jupiter (see Sovers and Fanselow (1987) page 9). Since this simple correction, suggested by Sovers and Fanselow (1987) and Hellings (1986) among others, is sufficient at the picosecond level, it was adapted for the consensus model. It is also necessary to account for the motion of station 2 during the propagation time between station 1 and station 2. In this model $\vec{R}_{i_J}$, the vector from the J^{th} gravitating body to the i^{th} receiver, is iterated once, giving

$$t_{1_J} = \min\left[t_1, t_1 - \frac{\hat{K}\cdot(\vec{X}_J(t_1) - \vec{X}_1(t_1))}{c}\right], \tag{3}$$

so that

$$\vec{R}_{1_J}(t_1) = \vec{X}_1(t_1) - \vec{X}_J(t_{1_J}), \tag{4}$$

and

$$\vec{R}_{2_J} = \vec{X}_2(t_1) - \frac{\vec{V}_\oplus}{c}(\hat{K}\cdot\vec{b}) - \vec{X}_J(t_{1_J}). \tag{5}$$

Only this one iteration is needed to obtain picosecond level accuracy for solar system objects. $\vec{X}_1(t_1)$ is not tabulated, but can be inferred from $\vec{X}_\oplus(t_1)$ using

$$\vec{X}_i(t_1) = \vec{X}_\oplus(t_1) + \vec{x}_i(t_1), \tag{6}$$

which is of sufficient accuracy for use in equations 3, 4, and 5, when substituted into equation 1 but not for use in computing the geometric delay. The total gravitational delay is the sum over all gravitating bodies including the Earth,

$$\Delta T_{grav} = \sum_J \Delta T_{grav_J}. \tag{7}$$

(b) Geometric Delay

In the barycentric frame the vacuum delay equation is, to a sufficient level of approximation:

$$T_2 - T_1 = -\frac{1}{c}\hat{K}\cdot(\vec{X}_2(T_2) - \vec{X}_1(T_1)) + \Delta T_{grav}. \tag{8}$$

This equation is converted into a geocentric delay equation using known quantities by performing the relativistic transformations relating the barycentric vectors $\vec{X}_i$ to the corresponding geocentric vectors $\vec{x}_i$, thus converting equation 8 into an equation in terms of $\vec{x}_i$. The related transformation between barycentric and geocentric time can be used to derive another equation relating $T_2 - T_1$ and $t_2 - t_1$, and these two equations can then be solved for the geocentric delay in terms of the geocentric baseline vector $\vec{b}$. In the rational polynomial form the total geocentric vacuum delay is given by

$$t_{v_2} - t_{v_1} = \frac{\Delta T_{grav} - \frac{\hat{K}\cdot\vec{b}}{c}\left[1 - \frac{(1+\gamma)U}{c^2} - \frac{|\vec{V}_\oplus|^2}{2c^2} - \frac{\vec{V}_\oplus\cdot\vec{w}_2}{c^2}\right] - \frac{\vec{V}_\oplus\cdot\vec{b}}{c^2}(1 + \hat{K}\cdot\vec{V}_\oplus/2c)}{1 + \frac{\hat{K}\cdot(\vec{V}_\oplus + \vec{w}_2)}{c}}. \tag{9}$$

Given this expression for the vacuum delay, the total delay is found to be

$$t_2 - t_1 = t_{v_2} - t_{v_1} + (\delta t_{atm_2} - \delta t_{atm_1}) + \delta t_{atm_1} \frac{\hat{K} \cdot (\vec{w}_2 - \vec{w}_1)}{c}. \tag{10}$$

For convenience the total delay can be divided into separate geometric and propagation delays. The geometric delay is given by

$$t_{g_2} - t_{g_1} = t_{v_2} - t_{v_1} + \delta t_{atm_1} \frac{\hat{K} \cdot (\vec{w}_2 - \vec{w}_1)}{c}, \tag{11}$$

and the total delay can be found at some later time by adding the propagation delay:

$$t_2 - t_1 = t_{g_2} - t_{g_1} + (\delta t_{atm_2} - \delta t_{atm_1}). \tag{12}$$

The tropospheric propagation delay in equations 11 and 12 need not be from the same model. The estimate in equation 12 should be as accurate as possible, while the δt_{atm} model in equation 11 need only be accurate to about an air mass ($\sim$ 10 nanoseconds). If equation 10 is used instead, the model should be as accurate as is possible. Note that the tropospheric delay is computed in the rest frame of each station and can be directly added to the geocentric delay (equation 11), at the uncertainty level considered here (see Eubanks, 1991; Treuhaft and Thomas, 1991).

If $\delta\vec{b}$ is the difference between the *a priori* baseline vector and the true baseline, the true delay may be computed from the *a priori* delay as follows. If $\delta\vec{b}$ is less than roughly three meters, then it suffices to add $-(\hat{K} \cdot \delta\vec{b})/c$ to the *a priori* delay. If this is not the case, however, the *a priori* delay must be modified by adding

$$\Delta(t_{g_2} - t_{g_1}) = -\frac{\frac{\hat{K} \cdot \delta\vec{b}}{c}}{1 + \frac{\hat{K} \cdot (\vec{V}_\oplus + \vec{w}_2)}{c}} - \frac{\vec{V}_\oplus \cdot \delta\vec{b}}{c^2}. \tag{13}$$

(c) Observations Close to the Sun

For observations made very close to the Sun, higher order relativistic time delay effects become increasingly important. The largest correction is due to the change in delay caused by the bending of the ray path by the gravitating body described in Richter and Matzner (1983) and Hellings (1986). The change to ΔT_{grav} is

$$\delta T_{grav_i} = \frac{4G^2 M_i^2}{c^5} \frac{\vec{b} \cdot (\hat{N}_{1_i} + \hat{K})}{(|\vec{R}|_{1_i} + \vec{R}_{1_i} \cdot \hat{K})^2}, \tag{14}$$

which should be added to the ΔT_{grav} in equation 1.

(d) Summary

Assuming that the reference time is the UTC time arrival of the VLBI signal at receiver 1, and that it is transformed to the appropriate timescale to be used to compute each element of the geometric model, the following steps are recommended to compute the VLBI time delay.

1. Use equation 6 to estimate the barycentric station vector for receiver 1.
2. Use equations 3, 4, and 5 to estimate the vectors from the Sun, the Moon, and each planet except the Earth to receiver 1.
3. Use equation 1 to estimate the differential gravitational delay for each of those bodies.
4. Use equation 2 to find the differential gravitational delay due to the Earth.
5. Sum to find the total differential gravitational delay.
6. Compute the vacuum delay from equation 9.
7. Calculate the aberrated source vector for use in the calculation of the tropospheric propagation delay:

$$\vec{k}_i = \hat{K} + \frac{\vec{V}_\oplus + \vec{w}_i}{c} - \hat{K}\frac{\hat{K} \cdot (\vec{V}_\oplus + \vec{w}_i)}{c}. \tag{15}$$

8. Add the geometric part of the tropospheric propagation delay to the vacuum delay, equation 11.
9. The total delay can be found by adding the best estimate of the tropospheric propagation delay
$$t_2 - t_1 = t_{g_2} - t_{g_1} + [\delta t_{atm_2}(t_1 - \frac{\hat{K}\cdot\vec{b}}{c}, \vec{k}_2) - \delta t_{atm_1}(\vec{k}_1)]. \tag{16}$$
10. If necessary, apply equation 13 to correct for "post-model" changes in the baseline by adding equation 13 to the total time delay from equation step 9.

11.2 Laser Ranging

In a reference system centered on an ensemble of masses, if a light signal is emitted from x_1 at coordinate time t_1 and is received at x_2 at coordinate time t_2, the coordinate time of propagation is given by
$$t_2 - t_1 = \frac{|\vec{x}_2(t_2) - \vec{x}_1(t_1)|}{c} + \sum_J \frac{2GM_J}{c^3} \ln\left(\frac{r_{J1} + r_{J2} + \rho}{r_{J1} + r_{J2} - \rho}\right), \tag{17}$$

where the sum is carried out over all bodies J with mass M_J centered at x_J and where $r_{J1} = |\vec{x}_1 - \vec{x}_J|$, $r_{J2} = |\vec{x}_2 - \vec{x}_J|$ and $\rho = |\vec{x}_2 - \vec{x}_1|$.

For near-Earth satellites (SLR), practical analysis is done in the geocentric frame of reference, and the only body to be considered is the Earth (Ries *et al.*, 1988). For lunar laser ranging (LLR), which is formulated in the solar system barycentric reference frame, the Sun and the Earth must be taken into account, with the contribution of the Moon being of order 1 ps (*i.e.* about 1 mm for a return trip). Moreover, in the analysis of LLR data, the body-centered coordinates of an Earth station and a lunar reflector should be transformed into barycentric coordinates. The transformation of $\vec{r}$, a geocentric position vector expressed in the GCRS, to $\vec{r}_b$, the vector expressed in the BCRS, is provided with an uncertainty lower than 1 mm by the equation

$$\vec{r}_b = \vec{r}\left(1 - \frac{U}{c^2}\right) - \frac{1}{2}\left(\frac{\vec{V}\cdot\vec{r}}{c^2}\right)\vec{V}, \tag{18}$$

where U is the gravitational potential at the geocenter (excluding the Earth's mass) and $\vec{V}$ is the barycentric velocity of the Earth. A similar equation applies to the selenocentric reflector coordinates.

In general, however, the geocentric and barycentric systems are chosen so that the geocentric space coordinates are "consistent with TT" (position vector $\vec{r}_{TT}$) and that the barycentric space coordinates are "consistent with TDB" (position vector $\vec{r}_{TDB}$). The transformation of $\vec{r}_{TT}$ to $\vec{r}_{TDB}$, is then given by

$$\vec{r}_{TDB} = \vec{r}_{TT}\left(1 - \frac{U}{c^2} - L_C\right) - \frac{1}{2}\left(\frac{\vec{V}\cdot\vec{r}_{TT}}{c^2}\right)\vec{V}, \tag{19}$$

where L_C is given in Table 1.1.

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