5 Transformation Between the Celestial and Terrestrial Systems (16 February 2007)

The coordinate transformation to be used to transform from the terrestrial reference system (TRS) to the celestial reference system (CRS) at the epoch t of the observation can be written as:

$$[\mathrm{CRS}] = Q(t)R(t)W(t)\ [\mathrm{TRS}], \tag{1}$$

where $Q(t)$, $R(t)$ and $W(t)$ are the transformation matrices arising from the motion of the celestial pole in the celestial system, from the rotation of the Earth around the axis of the pole, and from polar motion respectively. The frame as realized from the [TRS] by applying the transformations $W(t)$ and then $R(t)$ will be called "the intermediate reference frame of epoch t."

5.1 The Framework of IAU 2000 Resolutions

Several resolutions were adopted by the XXIVth General Assembly of the International Astronomical Union (Manchester, August 2000) that concern the transformation between the celestial and terrestrial reference systems and are therefore to be implemented in the IERS procedures. Such a transformation being also required for computing directions of celestial objects in intermediate systems, the process to transform among these systems consistent with the IAU resolutions is also provided at the end of this chapter.

Resolution B1.3 specifies that the systems of space-time coordinates as defined by IAU Resolution A4 (1991) for the solar system and the Earth within the framework of General Relativity are now named the Barycentric Celestial Reference System (BCRS) and the Geocentric Celestial Reference System (GCRS) respectively. It also provides a general framework for expressing the metric tensor and defining coordinate transformations at the first post-Newtonian level.

Resolution B1.6 recommends that, beginning on 1 January 2003, the IAU 1976 precession model and IAU 1980 theory of nutation be replaced by the precession-nutation model IAU 2000A (MHB 2000 based on the transfer functions of Mathews *et al.*, (2002)) for those who need a model at the 0.2 mas level, or its shorter version IAU 2000B for those who need a model only at the 1 mas level, together with their associated celestial pole offsets, published in this document.

Resolution B1.7 recommends that the Celestial Intermediate Pole (CIP) be implemented in place of the Celestial Ephemeris Pole (CEP) on 1 January 2003 and specifies how to implement its definition through its direction at J2000.0 in the GCRS as well as the realization of its motion both in the GCRS and ITRS. Its definition is an extension of that of the CEP in the high frequency domain and coincides with that of the CEP in the low frequency domain (Capitaine, 2000).

Resolution B1.8 recommends the use of the "non-rotating origin" (NRO) (Guinot, 1979) both in the GCRS and the ITRS and these origins are designated as the Celestial Intermediate Origin (CIO) and the Terrestrial Intermediate Origin (TIO). The "Earth Rotation Angle" is defined as the angle measured along the equator of the CIP between the CIO and the TIO. This resolution recommends that UT1 be linearly proportional to the Earth Rotation Angle and that the transformation between the ITRS and GCRS be specified by the position of the CIP in the GCRS, the position of the CIP in the ITRS, and the Earth Rotation Angle. It is recommended that the IERS takes steps to implement this by 1 January 2003 and that the IERS will continue to provide users with data and algorithms for the conventional transformation.

Following the recommendations above, this chapter of the IERS Conventions provides the expressions for the implementation of the IAU resolutions using the new transformation which is described in Resolution B1.8. It also provides the expressions which are necessary to be compatible with the resolutions when using the conventional transformation. Numerical values contained in this chapter have been slightly revised from earlier provisional values to ensure continuity of the IERS products. Fortran subroutines implementing the transformations are described towards the end of the chapter. More detailed explanations about the relevant concepts, software and IERS products can be found in IERS Technical Note 29 (Capitaine *et al.*, 2002).

5.2 Implementation of IAU 2000 Resolutions

In order to follow Resolution B1.3, the celestial reference system, which is designated here CRS, must correspond to the geocentric space coordinates of the GCRS. IAU Resolution A4 (1991) specified that the relative orientation of barycentric and geocentric spatial axes in BCRS and GCRS are without any time dependent rotation. This requires that the geodesic precession and nutation be taken into account in the precession-nutation model.

Concerning the time coordinates, IAU Resolution A4 (1991) defined TCB and TCG of the BCRS and GCRS respectively, as well as another time coordinate in the GCRS, Terrestrial Time (TT), which is the theoretical counterpart of the realized time scale TAI + 32.184 s and has been redefined by IAU resolution B1.9 (2000). See Chapter 10 for the relationships between these time scales.

The parameter t, used in the following expressions, is defined by

$$t = (\mathrm{TT} - 2000 \text{ January 1d 12h TT}) \text{ in days}/36525. \tag{2}$$

This definition is consistent with IAU Resolution C7 (1994) which recommends that the epoch J2000.0 be defined at the geocenter and at the date 2000 January 1.5 TT = Julian Date 2451545.0 TT.

In order to follow Resolution B1.6, the precession-nutation quantities to be used in the transformation matrix Q(t) must be based on the precession-nutation model IAU 2000A or IAU 2000B depending on the required precision. In order to follow Resolution B1.7, the realized celestial pole must be the CIP. This requires an offset at epoch in the conventional model for precession-nutation as well as diurnal and higher frequency variations in the Earth's orientation. According to this resolution, the direction of the CIP at J2000.0 has to be offset from the pole of the GCRS in a manner consistent with the IAU 2000A precession-nutation model. The motion of the CIP in the GCRS is realized by the IAU 2000 model for precession and forced nutation for periods greater than two days plus additional time-dependent corrections provided by the IERS through appropriate astro-geodetic observations. The motion of the CIP in the ITRS is provided by the IERS through astro-geodetic observations and models including variations with frequencies outside the retrograde diurnal band.

The realization of the CIP thus requires that the IERS monitor the observed differences (reported as "celestial pole offsets") with respect to the conventional celestial position of the CIP in the GCRS based on the IAU 2000 precession-nutation model together with its observed offset at epoch. It also requires that the motion of the CIP in the TRS be provided by the IERS by observations taking into account a predictable part specified by a model including the terrestrial motion of the pole corresponding to the forced nutations with periods less than two days (in the GCRS) as well as the tidal variations in polar motion. Two equivalent procedures were given in the IERS Conventions (McCarthy, 1996) for the coordinate transformation from the TRS to the CRS. The classical procedure, which was described in detail as option 1, makes use of the equinox for realizing the intermediate reference frame of date t. It uses apparent Greenwich Sidereal Time (GST) in the transformation matrix $R(t)$ and the classical precession and nutation parameters in the transformation matrix $Q(t)$.

The second procedure, which was described in detail as option 2, makes use of the "non-rotating origin" to realize the intermediate reference frame of date t. It uses the "Earth Rotation Angle," originally referred to as "stellar angle" in the transformation matrix $R(t)$, and the two coordinates of the celestial pole in the CRS (Capitaine, 1990) in the transformation matrix $Q(t)$.

Resolutions B1.3, B1.6 and B1.7 can be implemented in any of these procedures if the requirements described above are followed for the space-time coordinates in the geocentric celestial system, for the precession and nutation model on which are based the precession and nutation quantities used in the transformation matrix $Q(t)$ and for the polar motion used in the transformation matrix $W(t)$.

On the other hand, only the second procedure can be in agreement with Resolution B1.8, which requires the use of the "non-rotating origin" in both the CRS and the TRS as well as the position of the CIP in the GCRS and in the ITRS. However, the IERS must also provide users with data and algorithms for the conventional transformation; this implies that the expression of Greenwich Sidereal Time (GST) has to be consistent with the new procedure.

The following sections give the details of this procedure and the standard expressions necessary to obtain the numerical values of the relevant parameters at the date of the observation.

5.3 Coordinate Transformation consistent with the IAU 2000 Resolutions

In the following, R_1, R_2 and R_3 denote rotation matrices with positive angle about the axes 1, 2 and 3 of the coordinate frame. The position of the CIP both in the TRS and CRS is provided by the x and y components of the CIP unit vector. These components are called "coordinates" in the following and their numerical expressions are multiplied by the factor $1296000''/2\pi$ in order to provide in arcseconds the value of the corresponding "angles" with respect to the polar axis of the reference system.

The coordinate transformation (1) from the TRS to the CRS corresponding to the procedure consistent with Resolution B1.8 is expressed in terms of the three fundamental components as given below (Capitaine, 1990)

$$W(t) = R_3(-s') \cdot R_2(x_p) \cdot R_1(y_p), \tag{3}$$

x_p and y_p being the "polar coordinates" of the Celestial Intermediate Pole (CIP) in the TRS and s' being a quantity which provides the position of the TIO on the equator of the CIP corresponding to the kinematical definition of the NRO in the ITRS when the CIP is moving with respect to the ITRS due to polar motion. The expression of s' as a function of the coordinates x_p and y_p is:

$$s'(t) = (1/2) \int_{t_0}^{t} (x_p \dot{y}_p - \dot{x}_p y_p)\ dt. \tag{4}$$

The use of the quantity s', which was neglected in the classical form prior to 1 January 2003, is necessary to provide an exact realization of the "instantaneous prime meridian."

$$R(t) = R_3(-\theta), \tag{5}$$

θ being the Earth Rotation Angle between the CIO and the TIO at date t on the equator of the CIP, which provides a rigorous definition of the sidereal rotation of the Earth.

$$Q(t) = R_3(-E) \cdot R_2(-d) \cdot R_3(E) \cdot R_3(s), \tag{6}$$

E and d being such that the coordinates of the CIP in the CRS are:

$$X = \sin d \cos E, \qquad Y = \sin d \sin E, \qquad Z = \cos d, \tag{7}$$

and s being a quantity which provides the position of the CIO on the equator of the CIP corresponding to the kinematical definition of the NRO in the GCRS when the CIP is moving with respect to the GCRS, between the reference epoch and the epoch t due to precession and nutation. Its expression as a function of the coordinates X and Y is (Capitaine *et al.*, 2000)

$$s(t) = -\int_{t_0}^{t} \frac{X(t)\dot{Y}(t) - Y(t)\dot{X}(t)}{1 + Z(t)} dt - (\sigma_0 N_0 - \Sigma_0 N_0), \tag{8}$$

where σ_0 and Σ_0 are the positions of the CIO at J2000.0 and the x-origin of the GCRS respectively and N_0 is the ascending node of the equator at J2000.0 in the equator of the GCRS. Or equivalently, within 1 microarcsecond over one century

$$s(t) = -\frac{1}{2}[X(t)Y(t) - X(t_0)Y(t_0)] + \int_{t_0}^{t} \dot{X}(t)Y(t)dt - (\sigma_0 N_0 - \Sigma_0 N_0). \tag{9}$$

The arbitrary constant $\sigma_0 N_0 - \Sigma_0 N_0$, which had been conventionally chosen to be zero in previous references (*e.g.* Capitaine *et al.*, 2000), is now chosen to ensure continuity with the classical procedure on 1 January 2003 (see expression (36)).

$Q(t)$ can be given in an equivalent form directly involving X and Y as

$$Q(t) = \begin{pmatrix} 1 - aX^2 & -aXY & X \\ -aXY & 1 - aY^2 & Y \\ -X & -Y & 1 - a(X^2 + Y^2) \end{pmatrix} \cdot R_3(s), \tag{10}$$

with $a = 1/(1+\cos d)$, which can also be written, with an accuracy of 1 μas, as $a = 1/2 + 1/8(X^2 + Y^2)$. Such an expression of the transformation (1) leads to very simple expressions of the partial derivatives of observables with respect to the terrestrial coordinates of the CIP, UT1, and celestial coordinates of the CIP.

5.4 Parameters to be used in the Transformation

5.4.1 Schematic Representation of the Motion of the CIP

According to Resolution B1.7, the CIP is an intermediate pole separating, by convention, the motion of the pole of the TRS in the CRS into two parts:

- the celestial motion of the CIP (precession/nutation), including all the terms with periods greater than 2 days in the CRS (*i.e.* frequencies between -0.5 counts per sidereal day (cpsd) and $+0.5$ cpsd),
- the terrestrial motion of the CIP (polar motion), including all the terms outside the retrograde diurnal band in the TRS (*i.e.* frequencies lower than -1.5 cpsd or greater than -0.5 cpsd).

frequency in TRS	-3.5	-2.5	-1.5	-0.5	$+0.5$	$+1.5$	$+2.5$	(cpsd)
	------ *polar motion* |			| *polar motion* ------				
frequency in CRS	-2.5	-1.5	-0.5	$+0.5$	$+1.5$	$+2.5$	$+3.5$	(cpsd)
			|*nutation*|					

5.4.2 Motion of the CIP in the ITRS

The standard pole coordinates to be used for the parameters x_p and y_p, if not estimated from the observations, are those published by the IERS with additional components to account for the effects of ocean tides and for nutation terms with periods less than two days.

$$(x_p, y_p) = (x, y)_{IERS} + (\Delta x, \Delta y)_{tidal} + (\Delta x, \Delta y)_{nutation},$$

where $(x, y)_{IERS}$ are pole coordinates provided by the IERS, $(\Delta x, \Delta y)_{tidal}$ are the tidal components, and $(\Delta x, \Delta y)_{nutation}$ are the nutation components. The corrections for these variations are described below.

Corrections $(\Delta x, \Delta y)_{tidal}$ for the diurnal and sub-diurnal variations in polar motion caused by ocean tides can be computed using a routine available on the website of the IERS Conventions (see Chapter 8). Table 8.2 (from Ch. Bizouard), based on this routine, provides the amplitudes and arguments of these variations for the 71 tidal constituents considered in the model. These subdaily variations are not part of the polar motion values reported to and distributed by the IERS and are therefore to be added after interpolation.

Recent models for rigid Earth nutation (Souchay and Kinoshita, 1997; Bretagnon *et al.*, 1997; Folgueira *et al.*, 1998a; Folgueira *et al.*, 1998b; Souchay *et al.*, 1999; Roosbeek, 1999; Bizouard *et al.*, 2000; Bizouard *et al.*, 2001) include prograde diurnal and prograde semidiurnal terms with respect to the GCRS with amplitudes up to $\sim$ 15 μas in $\Delta\psi \sin\epsilon_0$ and $\Delta\epsilon$. The semidiurnal terms in nutation have also been provided both for rigid and nonrigid Earth models based on Hamiltonian formalism (Getino *et al.*, 2001, Escapa *et al.*, 2002a and b). In order to realize the

CIP as recommended by Resolution B1.7, nutations with periods less than two days are to be considered using a model for the corresponding motion of the pole in the ITRS. The prograde diurnal nutations correspond to prograde and retrograde long periodic variations in polar motion, and the prograde semidiurnal nutations correspond to prograde diurnal variations in polar motion (see for example Folgueira *et al.* 2001). A table for operational use of the model for these variations $(\Delta x, \Delta y)_{nutation}$ in polar motion for a nonrigid Earth has been provided by an *ad hoc* Working Group (Brzeziński, 2002) based on nonrigid Earth models and developments of the tidal potential (Brzeziński, 2001; Brzeziński and Capitaine, 2002; Mathews and Bretagnon, 2002). The amplitudes of the diurnal terms are in very good agreement with those estimated by Getino *et al.* (2001). Components with amplitudes greater than 0.5 μas are given in Table 5.1. The contribution from the triaxiality of the core to the diurnal waves, while it can exceed the adopted cut-off level (Escapa *et al.*, 2002b; Mathews and Bretagnon, 2002), has not been taken into account in the table due to the large uncertainty in the triaxiality of the core (Dehant, 2002, private communication). The Stokes coefficients of the geopotential are from the model JGM-3.

The diurnal components of these variations (namely, the 10 terms listed in Table 5.1 with periods near 1 day) should be considered similarly to the diurnal and semidiurnal variations due to ocean tides (see Chapter 8). They are not part of the polar motion values reported to the IERS and distributed by the IERS and should therefore be added after interpolation. The diurnal components $(\Delta x, \Delta y)_{nutation}$ can be computed with the routine PMsdnut.for, provided by Aleksander Brzeziński, available on the Conventions Center website <[1]>. The long-periodic terms, as well as the secular variation, are already contained in the observed polar motion and need not be added to the reported values.

5.4.3 Position of the TIO in the ITRS

The quantity s' is only sensitive to the largest variations in polar motion. Some components of s' have to be evaluated, in principle, from the measurements and can be extrapolated using the IERS data. Its main component can be written as:

$$s' = -0.0015(a_c^2/1.2 + a_a^2)t, \tag{11}$$

a_c and a_a being the average amplitudes (in arc seconds) of the Chandlerian and annual wobbles, respectively in the period considered (Capitaine *et al.*, 1986). The value of s' will therefore be less than 0.4 mas through the next century, even if the amplitudes for the Chandlerian and annual wobbles reach values of the order of $0.5''$ and $0.1''$ respectively. Using the current mean amplitudes for the Chandlerian and annual wobbles gives (Lambert and Bizouard, 2002):

$$s' = -47\ \mu\text{as}\ t. \tag{12}$$

5.4.4 Earth Rotation Angle

The Earth Rotation Angle, θ, is obtained by the use of its conventional relationship with UT1 as given by Capitaine *et al.* (2000),

$$\theta(T_u) = 2\pi(0.7790572732640 + 1.00273781191135448 T_u), \tag{13}$$

where T_u=(Julian UT1 date−2451545.0), and UT1=UTC+(UT1−UTC), or equivalently (modulo 2π), in order to reduce possible rounding errors,

$$\begin{aligned}\theta(T_u) \quad = \quad & 2\pi(\text{UT1 Julian day fraction} \\ & +0.7790572732640 + 0.00273781191135448 T_u),\end{aligned} \tag{14}$$

the quantity UT1−UTC to be used (if not estimated from the observations) being the IERS value. Note, that for 0^h UT1, the UT1 Julian day fraction in (14) is 0.5.

This definition of UT1 based on the CIO is insensitive at the microarcsecond level to the precession-nutation model and to the observed celestial pole offsets. Therefore, in the processing of observational data, the quantity s provided by Table 5.2c must be considered as independent of observations.

[1] ftp://tai.bipm.org/iers/convupdt/chapter5/

Table 5.1 Coefficients of sin(argument) and cos(argument) in $(\Delta x, \Delta y)_{nutation}$ due to tidal gravitation (of degree n) for a nonrigid Earth. Units are μas; χ denotes GMST+ π and the expressions for the fundamental arguments (Delaunay arguments) are given by (41).

		argument					Doodson	Period	x_p		y_p	
n	χ	l	l'	F	D	Ω	number	(days)	sin	cos	sin	cos
4	0	0	0	0	0	−1	055.565	6798.3837	−0.03	0.63	−0.05	−0.55
3	0	−1	0	1	0	2	055.645	6159.1355	1.46	0.00	−0.18	0.11
3	0	−1	0	1	0	1	055.655	3231.4956	−28.53	−0.23	3.42	−3.86
3	0	−1	0	1	0	0	055.665	2190.3501	−4.65	−0.08	0.55	−0.92
3	0	1	1	−1	0	0	056.444	438.35990	−0.69	0.15	−0.15	−0.68
3	0	1	1	−1	0	−1	056.454	411.80661	0.99	0.26	−0.25	1.04
3	0	0	0	1	−1	1	056.555	365.24219	1.19	0.21	−0.19	1.40
3	0	1	0	1	−2	1	057.455	193.55971	1.30	0.37	−0.17	2.91
3	0	0	0	1	0	2	065.545	27.431826	−0.05	−0.21	0.01	−1.68
3	0	0	0	1	0	1	065.555	27.321582	0.89	3.97	−0.11	32.39
3	0	0	0	1	0	0	065.565	27.212221	0.14	0.62	−0.02	5.09
3	0	−1	0	1	2	1	073.655	14.698136	−0.02	0.07	0.00	0.56
3	0	1	0	1	0	1	075.455	13.718786	−0.11	0.33	0.01	2.66
3	0	0	0	3	0	3	085.555	9.1071941	−0.08	0.11	0.01	0.88
3	0	0	0	3	0	2	085.565	9.0950103	−0.05	0.07	0.01	0.55
2	1	−1	0	−2	0	−1	135.645	1.1196992	−0.44	0.25	−0.25	−0.44
2	1	−1	0	−2	0	−2	135.655	1.1195149	−2.31	1.32	−1.32	−2.31
2	1	1	0	−2	−2	−2	137.455	1.1134606	−0.44	0.25	−0.25	−0.44
2	1	0	0	−2	0	−1	145.545	1.0759762	−2.14	1.23	−1.23	−2.14
2	1	0	0	−2	0	−2	145.555	1.0758059	−11.36	6.52	−6.52	−11.36
2	1	−1	0	0	0	0	155.655	1.0347187	0.84	−0.48	0.48	0.84
2	1	0	0	−2	2	−2	163.555	1.0027454	−4.76	2.73	−2.73	−4.76
2	1	0	0	0	0	0	165.555	0.9972696	14.27	−8.19	8.19	14.27
2	1	0	0	0	0	−1	165.565	0.9971233	1.93	−1.11	1.11	1.93
2	1	1	0	0	0	0	175.455	0.9624365	0.76	−0.43	0.43	0.76
Rate of secular polar motion (μas/y) due to the zero frequency tide												
4	0	0	0	0	0	0	555.555			−3.80		−4.31

5.4.5 Motion of the CIP in the GCRS

Developments of the coordinates X and Y of the CIP in the GCRS, valid at the microarcsecond level, based on the IERS 1996 model for precession, nutation and pole offset at J2000.0 with respect to the pole of the GCRS, have been provided by Capitaine *et al.* (2000). New developments of X and Y based on the IAU 2000A or IAU 2000B model (see the following section for more details) for precession-nutation and on their corresponding pole offset at J2000.0 with respect to the pole of the GCRS have been computed at the same accuracy (Capitaine *et al.*, 2003a). These developments have the following form:

$$
\begin{aligned}
X = \; & -0.01661699'' + 2004.19174288''t - 0.42721905''t^2 \\
& -0.19862054''t^3 - 0.00004605''t^4 + 0.00000598''t^5 \\
& + \textstyle\sum_i [(a_{\mathrm{s},0})_i \sin(\textsc{argument}) + (a_{\mathrm{c},0})_i \cos(\textsc{argument})] \\
& + \textstyle\sum_i [(a_{\mathrm{s},1})_i t \sin(\textsc{argument}) + (a_{\mathrm{c},1})_i t \cos(\textsc{argument})] \\
& + \textstyle\sum_i [(a_{\mathrm{s},2})_i t^2 \sin(\textsc{argument}) + (a_{\mathrm{c},2})_i t^2 \cos(\textsc{argument})] \\
& + \cdots ,
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
Y = \quad & -0.00695078'' - 0.02538199''t - 22.40725099''t^2 \\
& +0.00184228''t^3 + 0.00111306''t^4 + 0.00000099''t^5 \\
& + \textstyle\sum_i [(b_{\mathrm{c},0})_i \cos(\textsc{argument}) + (b_{\mathrm{s},0})_i \sin(\textsc{argument})] \\
& + \textstyle\sum_i [(b_{\mathrm{c},1})_i t \cos(\textsc{argument}) + (b_{\mathrm{s},1})_i \; t \sin(\textsc{argument})] \\
& + \textstyle\sum_i [(b_{\mathrm{c},2})_i t^2 \cos(\textsc{argument}) + (b_{\mathrm{s},2})_i t^2 \sin(\textsc{argument})] \\
& + \cdots ,
\end{aligned}
\tag{16}
$$

the parameter t being given by expression (2) and ARGUMENT being a function of the fundamental arguments of the nutation theory whose expressions are given by (41) for the lunisolar ones and (42) for the planetary ones.

These series are available electronically on the IERS Conventions Center website (Tables 5.2a and 5.2b) at <[2]>. `tab5.2a.txt` for the X coordinate and at `tab5.2b.txt` for the Y coordinate. An extract from Tables 5.2a and 5.2b for the largest non-polynomial terms in X and Y is given hereafter.

Extract from Tables 5.2a and 5.2b (available at < [2]>) for the largest non-polynomial terms in the development (15) for $X(t)$ (top part) and (16) for $Y(t)$ (bottom part) compatible with IAU 2000A precession-nutation model (unit μas). The expressions for the fundamental arguments appearing in columns 4 to 17 are given by (41) and (42).

i	$(a_{s,0})_i$	$(a_{c,0})_i$	l	l'	F	D	Ω	L_{Me}	L_{Ve}	L_E	L_{Ma}	L_J	L_{Sa}	L_U	L_{Ne}	p_A
1	−6844318.44	1328.67	0	0	0	0	1	0	0	0	0	0	0	0	0	0
2	−523908.04	−544.76	0	0	2	−2	2	0	0	0	0	0	0	0	0	0
3	−90552.22	111.23	0	0	2	0	2	0	0	0	0	0	0	0	0	0
4	82168.76	−27.64	0	0	0	0	2	0	0	0	0	0	0	0	0	0
5	58707.02	470.05	0	1	0	0	0	0	0	0	0	0	0	0	0	0
.....																
i	$(a_{s,1})_i$	$(a_{c,1})_i$	l	l'	F	D	Ω	L_{Me}	L_{Ve}	L_E	L_{Ma}	L_J	L_{Sa}	L_U	L_{Ne}	p_A
1307	−3328.48	205833.15	0	0	0	0	1	0	0	0	0	0	0	0	0	0
1308	197.53	12814.01	0	0	2	−2	2	0	0	0	0	0	0	0	0	0
1309	41.19	2187.91	0	0	2	0	2	0	0	0	0	0	0	0	0	0
.....																
i	$(b_{s,0})_i$	$(b_{c,0})_i$	l	l'	F	D	Ω	L_{Me}	L_{Ve}	L_E	L_{Ma}	L_J	L_{Sa}	L_U	L_{Ne}	p_A
1	1538.18	9205236.26	0	0	0	0	1	0	0	0	0	0	0	0	0	0
2	−458.66	573033.42	0	0	2	−2	2	0	0	0	0	0	0	0	0	0
3	137.41	97846.69	0	0	2	0	2	0	0	0	0	0	0	0	0	0
4	−29.05	−89618.24	0	0	0	0	2	0	0	0	0	0	0	0	0	0
5	−17.40	22438.42	0	1	2	−2	2	0	0	0	0	0	0	0	0	0
.....																
i	$(b_{s,1})_i$	$(b_{c,1})_i$	l	l'	F	D	Ω	L_{Me}	L_{Ve}	L_E	L_{Ma}	L_J	L_{Sa}	L_U	L_{Ne}	p_A
963	153041.82	878.89	0	0	0	0	1	0	0	0	0	0	0	0	0	0
964	11714.49	−289.32	0	0	2	−2	2	0	0	0	0	0	0	0	0	0
965	2024.68	−50.99	0	0	2	0	2	0	0	0	0	0	0	0	0	0
.....																

The numerical values of the coefficients of the polynomial part of X and Y are derived from the development as a function of time of the precession in longitude and obliquity and pole offset at J2000.0 and the amplitudes $(a_{s,j})_i$, $(a_{c,j})_i$, $(b_{c,j})_i$,$(b_{s,j})_i$ for $j = 0, 1, 2, ...$ are derived from the amplitudes of the precession and nutation series. The amplitudes $(a_{s,0})_i$, $(b_{c,0})_i$ of the sine and cosine terms in X and Y respectively are equal to the amplitudes $A_i \times \sin\epsilon_0$ and B_i of the series for nutation in longitude $\times \sin\epsilon_0$ and obliquity, except for a few terms in each coordinate X and Y which contain a contribution from crossed-nutation effects. The coordinates X and Y contain Poisson terms in $t\sin$, $t\cos$, $t^2\sin$, $t^2\cos$, ... which originate from crossed terms between precession and nutation.

[2]ftp://tai.bipm.org/iers/conv2003/chapter5/ or ftp://maia.usno.navy.mil/conv2000/chapter5/

The contributions (in μas) to expressions (15) and (16) from the frame biases are

$$\begin{aligned} dX &= -16617.0 - 1.6\ t^2 + 0.7\ \cos\Omega, \\ dY &= -6819.2 - 141.9\ t + 0.5\ \sin\Omega, \end{aligned} \tag{17}$$

the first term in each coordinate being the contribution from the celestial pole offset at J2000.0 and the following ones from the equinox offset at J2000.0 also called "frame bias in right ascension."

The celestial coordinates of the CIP, X and Y, can also be obtained at each time t as a function of the precession and nutation quantities provided by the IAU 2000 precession-nutation model. The developments to be used for the precession quantities and for the nutation angles referred to the ecliptic of date are described in the following section and a subroutine is available for the computation.

The relationships between the coordinates X and Y and the precession-nutation quantities are (Capitaine, 1990):

$$\begin{aligned} X &= \bar{X} + \xi_0 - d\alpha_0\ \bar{Y}, \\ Y &= \bar{Y} + \eta_0 + d\alpha_0\ \bar{X}, \end{aligned} \tag{18}$$

where ξ_0 and η_0 are the celestial pole offsets at the basic epoch J2000.0 and $d\alpha_0$ the right ascension of the mean equinox of J2000.0 in the CRS. (See the numbers provided below in (19) and (28) for these quantities.)

The mean equinox of J2000.0 to be considered is not the "rotational dynamical mean equinox of J2000.0" as used in the past, but the "inertial dynamical mean equinox of J2000.0" to which the recent numerical or analytical solutions refer. The latter is associated with the ecliptic in the inertial sense, which is the plane perpendicular to the vector angular momentum of the orbital motion of the Earth-Moon barycenter as computed from the velocity of the barycenter relative to an *inertial frame*. The rotational equinox is associated with the ecliptic in the rotational sense, which is perpendicular to the vector angular momentum computed from the velocity referred to the *rotating* orbital plane of the Earth-Moon barycenter. (The difference between the two angular momenta is the angular momentum associated with the rotation of the orbital plane.) See Standish (1981) for more details. The numerical value for $d\alpha_0$ as derived from Chapront *et al.* (2002) to be used in expression (18) is

$$d\alpha_0 = (-0.01460 \pm 0.00050)''. \tag{19}$$

Quantities $\bar{X}$ and $\bar{Y}$ are given by:

$$\begin{aligned} \bar{X} &= \sin\omega \sin\psi, \\ \bar{Y} &= -\sin\epsilon_0 \cos\omega + \cos\epsilon_0 \sin\omega \cos\psi \end{aligned} \tag{20}$$

where ϵ_0 $(= 84381.448'')$ is the obliquity of the ecliptic at J2000.0, ω is the inclination of the true equator of date on the fixed ecliptic of epoch and ψ is the longitude, on the ecliptic of epoch, of the node of the true equator of date on the fixed ecliptic of epoch; these quantities are such that

$$\omega = \omega_A + \Delta\epsilon_1; \qquad \psi = \psi_A + \Delta\psi_1, \tag{21}$$

where ψ_A and ω_A are the precession quantities in longitude and obliquity (Lieske *et al.*, 1977) referred to the ecliptic of epoch and $\Delta\psi_1$, $\Delta\epsilon_1$ are the nutation angles in longitude and obliquity referred to the ecliptic of epoch. (See the numerical developments provided for the precession quantities in (30) and (31).) $\Delta\psi_1$, $\Delta\epsilon_1$ can be obtained from the nutation angles $\Delta\psi$, $\Delta\epsilon$ in longitude and obliquity referred to the ecliptic of date. The following formulation from Aoki and Kinoshita (1983) has been verified to provide an accuracy better than one microarcsecond after one century:

$$\begin{aligned} \Delta\psi_1 \sin\omega_A &= \Delta\psi \sin\epsilon_A \cos\chi_A - \Delta\epsilon \sin\chi_A, \\ \Delta\epsilon_1 &= \Delta\psi \sin\epsilon_A \sin\chi_A + \Delta\epsilon \cos\chi_A, \end{aligned} \tag{22}$$

ω_A and ϵ_A being the precession quantities in obliquity referred to the ecliptic of epoch and the ecliptic of date respectively and χ_A the planetary precession along the equator (Lieske *et al.*, 1977).

As VLBI observations have shown that there are deficiencies in the IAU 2000A precession-nutation model of the order of 0.2 mas (Mathews *et al.*, 2002), the IERS will continue to publish observed estimates of the corrections to the IAU 2000 precession-nutation model. The observed differences with respect to the conventional celestial pole position defined by the models are monitored and reported by the IERS as "celestial pole offsets." Such time dependent offsets from the direction of the pole of the GCRS must be provided as corrections δX and δY to the X and Y coordinates. These corrections can be related to the current celestial pole offsets $\delta\psi$ and $\delta\epsilon$ using the relationship (20) between X and Y and the precession-nutation quantities and (22) for the transformation from ecliptic of date to ecliptic of epoch. The relationship can be written with one microarcsecond accuracy for one century, for values of $\delta\psi$ and $\delta\epsilon$ lower than 1 mas:

$$\begin{aligned} \delta X &= \delta\psi \sin\epsilon_A + (\psi_A \cos\epsilon_0 - \chi_A)\delta\epsilon, \\ \delta Y &= \delta\epsilon - (\psi_A \cos\epsilon_0 - \chi_A)\delta\psi \sin\epsilon_A. \end{aligned} \tag{23}$$

These observed offsets include the contribution of the Free Core Nutation (FCN) described in sub-section 5.5.1 on the IAU 2000 precession-nutation model. Using these offsets, the corrected celestial position of the CIP is given by

$$X = X(\text{IAU } 2000) + \delta X, \qquad Y = Y(\text{IAU } 2000) + \delta Y. \tag{24}$$

This is practically equivalent to replacing the transformation matrix Q with the rotation

$$\tilde{Q} = \begin{pmatrix} 1 & 0 & \delta X \\ 0 & 1 & \delta Y \\ -\delta X & -\delta Y & 1 \end{pmatrix} Q_{\text{IAU}}, \tag{25}$$

where Q_{IAU} represents the $Q(t)$ matrix based on the IAU 2000 precession-nutation model.

5.4.6 Position of the CIO in the GCRS

The numerical development of s compatible with the IAU 2000A precession-nutation model as well as the corresponding celestial offset at J2000.0 has been derived in a way similar to that based on the IERS Conventions 1996 (Capitaine *et al.*, 2000). It results from the expression for s (8) using the developments of X and Y as functions of time given by (15) and (16) (Capitaine *et al.*, 2003a). The numerical development is provided for the quantity $s + XY/2$, which requires fewer terms to reach the same accuracy than a direct development for s.

The constant term for s, which was previously chosen so that $s(J2000.0) = 0$, has now been fitted (Capitaine *et al.*, 2003b) so as to ensure continuity of UT1 at the date of change (1 January 2003) consistent with the Earth Rotation Angle (ERA) relationship and the current VLBI procedure for estimating UT1 (see (36)).

The complete series for $s + XY/2$ with all terms larger than 0.1 μas is available electronically on the IERS Conventions Center website < [1]> at `tab5.2c.txt` and the terms larger than 0.5 μas over 25 years in the development of s are provided in Table 5.2c with microarcsecond accuracy.

5.5 IAU 2000A and IAU 2000B Precession-Nutation Model

5.5.1 Description of the Model

The IAU 2000A precession-nutation model has been adopted by the IAU (Resolution B1.6) to replace the IAU 1976 precession model (Lieske *et al.*, 1977) and the IAU 1980 theory of nutation (Wahr, 1981; Seidelmann, 1982). See Dehant *et al.* (1999) for more details. This model, developed by Mathews *et al.* (2002), is based on the solution of the linearized dynamical equation of the wobble-nutation problem and makes use of estimated values of seven of the parameters appearing in the theory, obtained from a least-squares fit of the theory to an up-to-date precession-nutation VLBI data set (Herring *et al.*, 2002). The nutation series relies on the Souchay *et al.* (1999) Rigid Earth nutation series, rescaled by 1.000012249 to account for the change in the dynamical ellipticity of the Earth implied by the observed correction to the lunisolar precession of the equator. The

Table 5.2c Development of $s(t)$ compatible with IAU 2000A precession-nutation model: all terms exceeding 0.5 μas during the interval 1975–2025 (unit μas).

$$s(t) = -XY/2 + 94 + 3808.35t - 119.94t^2 - 72574.09t^3 + \sum_k C_k \sin\alpha_k$$
$$+1.71t\sin\Omega + 3.57t\cos 2\Omega + 743.53t^2\sin\Omega + 56.91t^2\sin(2F-2D+2\Omega)$$
$$+9.84t^2\sin(2F+2\Omega) - 8.85t^2\sin 2\Omega$$

Argument α_k	Amplitude C_k
Ω	-2640.73
2Ω	-63.53
$2F-2D+3\Omega$	-11.75
$2F-2D+\Omega$	-11.21
$2F-2D+2\Omega$	$+4.57$
$2F+3\Omega$	-2.02
$2F+\Omega$	-1.98
3Ω	$+1.72$
$l'+\Omega$	$+1.41$
$l'-\Omega$	$+1.26$
$l+\Omega$	$+0.63$
$l-\Omega$	$+0.63$

nonrigid Earth transformation is the MHB2000 model of Mathews *et al.* (2002) which improves the IAU 1980 theory of nutation by taking into account the effect of mantle anelasticity, ocean tides, electromagnetic couplings produced between the fluid outer core and the mantle as well as between the solid inner core and fluid outer core (Buffett *et al.*, 2002) and the consideration of nonlinear terms which have hitherto been ignored in this type of formulation.

The resulting nutation series includes 678 lunisolar terms and 687 planetary terms which are expressed as "in-phase" and "out-of-phase" components with their time variations (see expression (29)). It provides the direction of the celestial pole in the GCRS with an accuracy of 0.2 mas. It includes the geodesic nutation contributions to the annual, semiannual and 18.6-year terms to be consistent with including the geodesic precession p_g in the precession model and so that the BCRS and GCRS are without any time-dependent rotation. The IAU 1976 precession model uses $p_g = 1.92''/c$ and the theoretical geodesic nutation contribution (Fukushima, 1991) used in the MHB model (Mathews *et al.*, 2002) is, in μas, for the nutations in longitude $\Delta\psi_g$ and obliquity $\Delta\epsilon_g$

$$\begin{aligned} \Delta\psi_g &= -153\ \sin l' - 2\ \sin 2l' + 3\ \sin\Omega, \\ \Delta\epsilon_g &= 1\ \cos\Omega, \end{aligned} \tag{26}$$

where l' is the mean anomaly of the Sun and Ω the longitude of the ascending node of the Moon. On the other hand, the FCN, being a free motion which cannot be predicted rigorously, is not considered a part of the IAU 2000A model.

The IAU 2000 nutation series is associated with improved numerical values for the precession rate of the equator in longitude and obliquity, which correspond to the following correction to the IAU 1976 precession:

$$\begin{aligned} \delta\psi_A &= (-0.29965 \pm 0.00040)''/\mathrm{c}, \\ \delta\omega_A &= (-0.02524 \pm 0.00010)''/\mathrm{c}, \end{aligned} \tag{27}$$

as well as with the following offset (originally provided as frame bias in dψ_{bias} and dϵ_{bias}) of the direction of the CIP at J2000.0 from the direction of the pole of the GCRS:

$$\begin{aligned} \xi_0 &= (-0.0166170 \pm 0.0000100)'', \\ \eta_0 &= (-0.0068192 \pm 0.0000100)''. \end{aligned} \tag{28}$$

The IAU 2000 nutation model is given by a series for nutation in longitude $\Delta\psi$ and obliquity $\Delta\epsilon$, referred to the ecliptic of date, with t measured in Julian centuries from epoch J2000.0:

$$\begin{aligned}\Delta\psi &= \textstyle\sum_{i=1}^{N}(A_i + A'_i t)\sin(\mathrm{ARGUMENT}) + (A''_i + A'''_i t)\cos(\mathrm{ARGUMENT}),\\ \Delta\epsilon &= \textstyle\sum_{i=1}^{N}(B_i + B'_i t)\cos(\mathrm{ARGUMENT}) + (B''_i + B'''_i t)\sin(\mathrm{ARGUMENT}).\end{aligned} \tag{29}$$

More details about the coefficients and arguments of these series (see extract of the Tables 5.3a and 5.3b below) will be given in section 5.8.

These series are available electronically on the IERS Conventions Center website < [1]>, for the lunisolar and planetary components respectively at `tab5.3a.txt` and `tab5.3b.txt`.

Extract from Tables 5.3a (lunisolar nutations, top part) and 5.3b (planetary nutations, bottom part) (available at < [1]>) providing the largest components for the "in-phase" and "out-of-phase" terms in longitude and obliquity. Units are mas and mas/c for the coefficients and their time variations respectively. Periods are in days.

l	l'	F	D	Ω	Period	A_i	A'_i	B_i	B'_i	A''_i	A'''_i	B''_i	B'''_i
0	0	0	0	1	-6798.383	-17206.4161	-17.4666	9205.2331	0.9086	3.3386	0.0029	1.5377	0.0002
0	0	2	-2	2	182.621	-1317.0906	-0.1675	573.0336	-0.3015	-1.3696	0.0012	-0.4587	-0.0003
0	0	2	0	2	13.661	-227.6413	-0.0234	97.8459	-0.0485	0.2796	0.0002	0.1374	-0.0001
0	0	0	0	2	-3399.192	207.4554	0.0207	-89.7492	0.0470	-0.0698	0.0000	-0.0291	0.0000
0	1	0	0	0	365.260	147.5877	-0.3633	7.3871	-0.0184	1.1817	-0.0015	-0.1924	0.0005
0	1	2	-2	2	121.749	-51.6821	0.1226	22.4386	-0.0677	-0.0524	0.0002	-0.0174	0.0000
1	0	0	0	0	27.555	71.1159	0.0073	-0.6750	0.0000	-0.0872	0.0000	0.0358	0.0000
0	0	2	0	1	13.633	-38.7298	-0.0367	20.0728	0.0018	0.0380	0.0001	0.0318	0.0000
1	0	2	0	2	9.133	-30.1461	-0.0036	12.9025	-0.0063	0.0816	0.0000	0.0367	0.0000
0	-1	2	-2	2	365.225	21.5829	-0.0494	-9.5929	0.0299	0.0111	0.0000	0.0132	-0.0001

														Period	Longitude		Obliquity	
l	l'	F	D	Ω	L_{Me}	L_{Ve}	L_E	L_{Ma}	L_J	L_{Sa}	L_U	L_{Ne}	p_A		A_i	A''_i	B''_i	B_i
0	0	1	-1	1	0	0	-1	0	-2	5	0	0	0	311921.26	-0.3084	0.5123	0.2735	0.1647
0	0	0	0	0	0	0	0	0	-2	5	0	0	1	311927.52	-0.1444	0.2409	-0.1286	-0.0771
0	0	0	0	0	0	-3	5	0	0	0	0	0	2	2957.35	-0.2150	0.0000	0.0000	0.0932
0	0	1	-1	1	0	-8	12	0	0	0	0	0	0	-88082.01	0.1200	0.0598	0.0319	-0.0641
0	0	0	0	0	0	0	0	0	2	0	0	0	2	2165.30	-0.1166	0.0000	0.0000	0.0505
0	0	0	0	0	0	0	4	-8	3	0	0	0	0	-651391.30	-0.0462	0.1604	0.0000	0.0000
0	0	0	0	0	0	1	-1	0	0	0	0	0	0	583.92	0.1485	0.0000	0.0000	0.0000
0	0	0	0	0	0	0	8	-16	4	5	0	0	0	34075700.82	0.1440	0.0000	0.0000	0.0000
0	0	0	0	0	0	0	1	0	-1	0	0	0	0	398.88	-0.1223	-0.0026	0.0000	0.0000
0	0	0	0	1	0	0	-1	2	0	0	0	0	0	37883.60	-0.0460	-0.0435	-0.0232	0.0246

The IAU2000A.f subroutine, provided by T. Herring, is available electronically on the IERS Conventions Center website at < [2]>. It produces the quantities to implement the IAU 2000A precession-nutation model based on the MHB 2000 model: nutation in longitude and obliquity, plus the contribution of the corrections to the IAU 1976 precession rates, plus the frame bias $d\psi_{bias}$ and $d\epsilon_{bias}$ in longitude and obliquity. The "total nutation" includes all components with the exception of the free core nutation (FCN). The software can also be used to model the expected FCN based on the most recent astronomical observations.

As recommended by Resolution B1.6, an abridged model, designated IAU 2000B, is available for those who need a model only at the 1 mas level. Such a model has been developed by McCarthy and Luzum (2003). It includes fewer than 80 lunisolar terms plus a bias to account for the effect of the planetary terms in the time period under consideration. It provides the celestial pole motion with an accuracy that does not result in a difference greater than 1 mas with respect to that of the IAU 2000A model during the period 1995–2050. The IAU2000B.f subroutine is available electronically on the IERS Conventions Center website at < [2]>.

5.5.2 Precession Developments compatible with the IAU2000 Model

The numerical values for the precession quantities compatible with the IAU 2000 precession-nutation model can be provided by using the developments (30) of Lieske *et al.* (1977) to which the estimated corrections (27) $\delta\psi_A$ and $\delta\omega_A$ to the IAU 1976 precession have to be added.

The expressions of Lieske *et al.* (1977) are

$$\begin{aligned}
\psi_A &= 5038.7784''t - 1.07259''t^2 - 0.001147''t^3, \\
\omega_A &= \epsilon_0 + 0.05127''t^2 - 0.007726''t^3, \\
\epsilon_A &= \epsilon_0 - 46.8150''t - 0.00059''t^2 + 0.001813''t^3, \\
\chi_A &= 10.5526''t - 2.38064''t^2 - 0.001125''t^3 \quad ,
\end{aligned} \tag{30}$$

and

$$\begin{aligned}
\zeta_A &= 2306.2181''t + 0.30188''t^2 + 0.017998''t^3, \\
\theta_A &= 2004.3109''t - 0.42665''t^2 - 0.041833''t^3, \\
z_A &= 2306.2181''t + 1.09468''t^2 + 0.018203''t^3,
\end{aligned} \tag{31}$$

with $\epsilon_0 = 84381.448''$.

Due to their theoretical bases, the original development of the precession quantities as function of time can be considered as being expressed in TDB.

The expressions compatible with the IAU 2000A precession and nutation are:

$$\begin{aligned}
\psi_A &= 5038.47875''t - 1.07259''t^2 - 0.001147''t^3, \\
\omega_A &= \epsilon_0 - 0.02524''t + 0.05127''t^2 - 0.007726''t^3, \\
\epsilon_A &= \epsilon_0 - 46.84024''t - 0.00059''t^2 + 0.001813''t^3, \\
\chi_A &= 10.5526''t - 2.38064''t^2 - 0.001125''t^3 \quad ,
\end{aligned} \tag{32}$$

and the following series has been developed (Capitaine *et al.*, 2003c) in order to match the 4-rotation series for precession $R_1(-\epsilon_0)\cdot R_3(\psi_A)\cdot R_1(\omega_A)\cdot R_3(-\chi_A)$, called the "canonical 4-rotation method," to sub-microarcsecond accuracy over 4 centuries:

$$\begin{aligned}
\zeta_A = {} & 2.5976176'' + 2306.0809506''t + 0.3019015''t^2 + 0.0179663''t^3 \\
& -0.0000327''t^4 - 0.0000002''t^5, \\
\theta_A = {} & 2004.1917476''t - 0.4269353''t^2 - 0.0418251''t^3 \\
& -0.0000601''t^4 - 0.0000001''t^5, \\
z_A = {} & -2.5976176'' + 2306.0803226''t + 1.0947790''t^2 + 0.0182273''t^3 \\
& +0.0000470''t^4 - 0.0000003''t^5.
\end{aligned} \tag{33}$$

Note that the new expression for the quantities ζ_A and z_A include a constant term (with opposite signs) which originates from the ratio between the precession rate in ω_A and in $\psi_A \sin\epsilon_0$.

In practice, TT is used in the above expressions in place of TDB. The largest term in the difference TDB$-$TT being 1.7 ms $\times \sin l'$, the resulting error in the precession quantity ψ_A is periodic, with an annual period and an amplitude of $2.7'' \times 10^{-9}$, which is significantly under the required microarcsecond accuracy.

5.6 Procedure to be used for the Transformation consistent with IAU 2000 Resolutions

There are several ways to implement the IAU 2000 precession-nutation model, and the precession developments to be used should be consistent with the procedure being used. The subroutines available for the different procedures are described below.

Using the new paradigm, the complete procedure to transform from the GCRS to the ITRS compatible with the IAU 2000A precession-nutation is based on the expressions provided by (15) and (16) and Tables 5.2 for the positions of the CIP and the CIO in the GCRS. These already contain the proper expressions for the new precession-nutation model and the frame biases. Another procedure can also be used for the computation of the coordinates X and Y of the CIP in the GCRS using expressions (18) to (22). This must be based on the MHB 2000 nutation series, on offsets at J2000.0 as well as on precession quantities $\psi_A, \omega_A, \epsilon_A, \chi_A$, taking into account the corrections to the IAU 1976 precession rates. (See expressions (32).)

In support of the classical paradigm, the IAU2000A subroutine provides the components of the precession-nutation model including the contributions of the correction to the IAU 1976 precession rates for ζ_A, θ_Z, z_A (see expressions (31)). Expressions (33) give the same angles but taking into account the IAU 2000 corrections.

The recommended option for implementing the IAU 2000A/B model using the classical transformation between the TRS and the GCRS is to follow a rigorous procedure described by Wallace (in Capitaine *et al.*, 2002). This procedure is composed of the classical nutation matrix using the MHB 2000 series, the precession matrix including four rotations $(R_1(-\epsilon_0) \cdot R_3(\psi_A) \cdot R_1(\omega_A) \cdot R_3(-\chi_A))$ using the updated developments (32) for these quantities and a separate rotation matrix for the frame bias.

In the case when one elects to continue using the classical expressions based on the IAU 1976 precession model and IAU 1980 theory of nutation, one should proceed as in the past as described in the IERS Conventions 1996 (McCarthy, 1996) and then apply the corrections to the model provided by the appropriate IAU 2000A/B software.

5.7 Expression of Greenwich Sidereal Time referred to the CIO

Greenwich Sidereal Time (GST), which refers to the equinox, is related to the "Earth Rotation Angle" ERA, denoted by θ, that refers to the Celestial Intermediate Origin (CIO), by the following relationship (Aoki and Kinoshita, 1983; Capitaine and Gontier, 1993) at the microarcsecond level:

$$\mathrm{GST} = dT_0 + \theta + \int_{t_0}^{t} (\dot{\widehat{\psi_A + \Delta\psi_1}}) \cos(\omega_A + \Delta\epsilon_1) dt - \chi_A + \Delta\psi \cos\epsilon_A - \Delta\psi_1 \cos\omega_A, \quad (34)$$

$\Delta\psi_1$, $\Delta\epsilon_1$, given by (22), being the nutation angles in longitude and obliquity referred to the ecliptic of epoch and χ_A, whose development is given in (32), the planetary precession along the equator (*i.e.* the RA component of the precession of the ecliptic).

This can be written as:

$$\mathrm{GST} = \theta(\mathrm{UT1}) - \mathrm{EO}, \quad (35)$$

where EO is "the equation of the origins" (according to the recommendations of the IAU WG on "Nomenclature for fundamental astronomy"), defined by:

$$\mathrm{EO} = -\ dT_0 - \int_{t_0}^{t} (\dot{\widehat{\psi_A + \Delta\psi_1}}) \cos(\omega_A - \Delta\epsilon_1) dt + \chi_A - \Delta\psi \cos\epsilon_A + \Delta\psi_1 \cos\omega_A, \quad (36)$$

providing the CIO right ascension of the equinox along the moving equator, which accounts for the accumulated precession and nutation in right ascension from J2000.0 to the epoch t, and dT_0 is a constant term to be fitted in order to ensure continuity in UT1 at the date of change. The numerical expression consistent with the IAU 2000A precession-nutation model has been obtained using computations similar to those performed for s and following a procedure, which is described below, to ensure consistency at the microarcsecond level among the transformations as well as continuity in UT1 at the date of change (Capitaine *et al.*, 2003b).

Table 5.4 Development of EO compatible with IAU 2000A precession-nutation model: all terms exceeding 0.5 μas during the interval 1975–2025 (unit μas).

$$\mathrm{EO} = -0.014506'' - 4612.15739966''t - 1.39667721''t^2 + 0.00009344''t^3 - \Delta\psi\cos\epsilon_A - \sum_k C'_k \sin\alpha_k$$

Argument α_k	Amplitude C'_k
Ω	+2640.96
2Ω	+63.52
$2F-2D+3\Omega$	+11.75
$2F-2D+\Omega$	+11.21
$2F-2D+2\Omega$	−4.55
$2F+3\Omega$	+2.02
$2F+\Omega$	+1.98
3Ω	−1.72
$l'+\Omega$	−1.41
$l'-\Omega$	−1.26
$l+\Omega$	−0.63
$l-\Omega$	−0.63

The series providing the expression for Greenwich Sidereal Time based on the IAU 2000A precession-nutation model is available on the IERS Conventions Center website < [1]> at `tab5.4.txt`, and the terms larger than 0.5 μas over 25 years in the development of EO are provided in Table 5.4 with microarcsecond accuracy.

The C'_k coefficients are similar to the C_k coefficients appearing in Table 5.2c providing the development for s and are equal to these coefficients up to 1 μas. The last term of EO, *i.e.* $-\sum_k C'_k \sin\alpha_k$, is the complementary term to be subtracted from the classical "equation of the equinoxes," $\Delta\psi\cos\epsilon_A$, to provide the relationship between GST and θ with microarcsecond accuracy. This replaces the two complementary terms provided in the *IERS Conventions 1996*. A secular term similar to that appearing in the quantity s is included in expression (36). This expression for GST used in the classical transformation based on the IAU 2000A precession-nutation ensures consistency at the microarcsecond level after one century among transformations using expressions (14) for θ, (15) and (16) for the celestial coordinates of the CIP and Table 5.2c for s. The numerical values for the constant term dT_0 in GST, which ensures continuity in UT1 at the date of change (1 January 2003), and for the corresponding constant term in s have been found to be

$$\begin{aligned} dT_0 &= +14506\,\mu\mathrm{as}, \\ s_0 &= +\ 94\,\mu\mathrm{as}. \end{aligned} \tag{37}$$

The change in the polynomial part of GST due to the correction in the precession rates (27) corresponds to a change dGMST (see also Williams, 1994) in the current relationship between GMST and UT1 (Aoki *et al.*, 1982). Its numerical expression derived from expressions (35) for GST, (13) for θ(UT1), and the polynomial part of EO in Table 5.4, minus the expression for GMST_{1982}(UT1), can be written in microarcseconds as

$$d\mathrm{GMST} = 14506 - 274950.12\,t + 117.21\,t^2 - 0.44\,t^3 + 18.82\,t^4. \tag{38}$$

The new expression for GST clearly distinguishes between θ, which is expressed as a function of UT1, and the EO (*i.e.* mainly the accumulated precession-nutation in right ascension), which is expressed in TDB (or, in practice, TT), in contrast to the GMST_{1982}(UT1) expression, which used only UT1. This gives rise to an additional difference in dGMST of (TT−UT1) multiplied by the speed of precession in right ascension. Using TT−TAI = 32.184 s, this is: [47+1.5(TAI−UT1)] μas, where TAI−UT1 is in seconds. On 1 January 2003, this difference is about 94 μas (see Gontier in Capitaine *et al.*, 2002), using the value of 32.3 s for TAI−UT1. This contribution for the effect of time scales is included in the values for dT_0 and s_0.

5.8 The Fundamental Arguments of Nutation Theory

5.8.1 The Multipliers of the Fundamental Arguments of Nutation Theory

Each of the lunisolar terms in the nutation series is characterized by a set of five integers N_j which determines the ARGUMENT for the term as a linear combination of the five Fundamental Arguments F_j, namely the Delaunay variables $(\ell, \ell', F, D, \Omega)$: ARGUMENT $= \sum_{j=1}^{5} N_j F_j$, where the values $(N_1, \cdots, N_5)$ of the multipliers characterize the term. The F_j are functions of time, and the angular frequency of the nutation described by the term is given by

$$\omega \equiv d(\mathrm{ARGUMENT})/dt. \tag{39}$$

The frequency thus defined is positive for most terms, and negative for some. Planetary nutation terms differ from the above only in that ARGUMENT $= \sum_{j=1}^{14} N'_j F'_j$, F_6 to F_{13}, as noted in Table 5.3, are the mean longitudes of the planets Mercury to Neptune including the Earth (l_{Me}, l_{Ve}, l_E, l_{Ma}, l_{Ju}, l_{Sa}, l_{Ur}, l_{Ne}) and F_{14} is the general precession in longitude p_a.

Over time scales involved in nutation studies, the frequency ω is effectively time-independent, and one may write, for the kth term in the nutation series,

$$\mathrm{ARGUMENT} = \omega_k t + \alpha_k. \tag{40}$$

Different tables of nutations in longitude and obliquity do not necessarily assign the same set of multipliers N_j to a particular term in the nutation series. The differences in the assignments arises from the fact that the replacement $(N_{j=1,14}) \rightarrow -(N_{j=1,14})$ accompanied by reversal of the sign of the coefficient of sin(ARGUMENT) in the series for $\Delta\psi$ and $\Delta\epsilon$ leaves these series unchanged.

5.8.2 Development of the Arguments of Lunisolar Nutation

The expressions for the fundamental arguments of nutation are given by the following developments where t is measured in Julian centuries of TDB (Simon *et al.*, 1994: Tables 3.4 (b.3) and 3.5 (b)) based on IERS 1992 constants and Williams *et al.* (1991) for precession.

$$\begin{aligned}
F_1 \equiv l \ &= Mean\ Anomaly\ of\ the\ Moon \\
&= 134.96340251^\circ + 1717915923.2178''t + 31.8792''t^2 \\
&\quad +0.051635''t^3 - 0.00024470''t^4, \\
F_2 \equiv l' \ &= Mean\ Anomaly\ of\ the\ Sun \\
&= 357.52910918^\circ + 129596581.0481''t - 0.5532''t^2 \\
&\quad +0.000136''t^3 - 0.00001149''t^4, \\
F_3 \equiv F &= L - \Omega \\
&= 93.27209062^\circ + 1739527262.8478''t - 12.7512''t^2 \\
&\quad -0.001037''t^3 + 0.00000417''t^4, \\
F_4 \equiv D &= Mean\ Elongation\ of\ the\ Moon\ from\ the\ Sun \\
&= 297.85019547^\circ + 1602961601.2090''t - 6.3706''t^2 \\
&\quad +0.006593''t^3 - 0.00003169''t^4, \\
F_5 \equiv \Omega &= Mean\ Longitude\ of\ the\ Ascending\ Node\ of\ the\ Moon \\
&= 125.04455501^\circ - 6962890.5431''t + 7.4722''t^2 \\
&\quad +0.007702''t^3 - 0.00005939''t^4
\end{aligned} \tag{41}$$

where L is the Mean Longitude of the Moon.

5.8.3 Development of the Arguments for the Planetary Nutation

Note that in the MHB 2000 code (IAU2000A.f) simplified expressions are used for the planetary nutation. The maximum difference in the nutation amplitudes is less than 0.1μas.

The mean longitudes of the planets used in the arguments for the planetary nutations are essentially those provided by Souchay *et al.* (1999), based on theories and constants of VSOP82 (Bretagnon, 1982) and ELP 2000 (Chapront-Touzé and Chapront, 1983) and developments of Simon *et al.* (1994). Their developments are given below in radians with t in Julian centuries.

The general precession, F_{14}, is from Kinoshita and Souchay (1990).

In the original expressions, t is measured in TDB. However, TT can be used in place of TDB as the difference due to TDB−TT is 0.9 mas $\times \sin l'$ for the largest effect in the nutation arguments, which produces a negligible difference (less than $10^{-2}\mu$as with a period of one year) in the corresponding amplitudes of nutation.

$$\begin{aligned}
F_6 &\equiv l_{Me} = 4.402608842 + 2608.7903141574 \times t, \\
F_7 &\equiv l_{Ve} = 3.176146697 + 1021.3285546211 \times t, \\
F_8 &\equiv l_E = 1.753470314 + 628.3075849991 \times t, \\
F_9 &\equiv l_{Ma} = 6.203480913 + 334.0612426700 \times t, \\
F_{10} &\equiv l_{Ju} = 0.599546497 + 52.9690962641 \times t, \\
F_{11} &\equiv l_{Sa} = 0.874016757 + 21.3299104960 \times t, \\
F_{12} &\equiv l_{Ur} = 5.481293872 + 7.4781598567 \times t, \\
F_{13} &\equiv l_{Ne} = 5.311886287 + 3.8133035638 \times t, \\
F_{14} &\equiv p_a = 0.02438175 \times t + 0.00000538691 \times t^2.
\end{aligned} \tag{42}$$

5.9 Prograde and Retrograde Nutation Amplitudes

The quantities $\Delta\psi(t)\sin\epsilon_0$ and $\Delta\epsilon(t)$ may be viewed as the components of a moving two-dimensional vector in the mean equatorial frame, with the positive X and Y axes pointing along the directions of increasing $\Delta\psi$ and $\Delta\epsilon$, respectively. The purely periodic parts of $\Delta\psi(t)\sin\epsilon_0$ and $\Delta\epsilon(t)$ for a term of frequency ω_k are made up of in-phase and out-of-phase parts

$$\begin{aligned}
(\Delta\psi^{ip}(t)\sin\epsilon_0,\ \Delta\epsilon^{ip}(t)) &= (\Delta\psi_k^{ip}\sin\epsilon_0\sin(\omega_k t + \alpha_k),\ \Delta\epsilon_k^{ip}\cos(\omega_k t + \alpha_k)), \\
(\Delta\psi^{op}(t)\sin\epsilon_0,\ \Delta\epsilon^{op}(t)) &= (\Delta\psi_k^{op}\sin\epsilon_0\cos(\omega_k t + \alpha_k),\ \Delta\epsilon_k^{op}\sin(\omega_k t + \alpha_k)),
\end{aligned} \tag{43}$$

respectively. Each of these vectors may be decomposed into two uniformly rotating vectors, one constituting a prograde circular nutation (rotating in the same sense as from the positive X axis towards the positive Y axis) and the other a retrograde one rotating in the opposite sense. The decomposition is facilitated by factoring out the sign q_k of ω_k from the argument, q_k being such that

$$q_k\omega_k \equiv |\omega_k|. \tag{44}$$

and writing

$$\omega_k t + \alpha_k = q_k(|\omega_k|t + q_k\alpha_k) \equiv q_k\chi_k, \tag{45}$$

with χ_k increasing linearly with time. The pair of vectors above then becomes

$$\begin{aligned}
(\Delta\psi^{ip}(t)\sin\epsilon_0,\ \Delta\epsilon^{ip}(t)) &= (q_k\Delta\psi_k^{ip}\sin\epsilon_0\sin\chi_k,\ \Delta\epsilon_k^{ip}\cos\chi_k), \\
(\Delta\psi^{op}(t)\sin\epsilon_0,\ \Delta\epsilon^{op}(t)) &= (\Delta\psi_k^{op}\sin\epsilon_0\cos\chi_k,\ q_k\Delta\epsilon_k^{op}\sin\chi_k).
\end{aligned} \tag{46}$$

Because χ_k increases linearly with time, the mutually orthogonal unit vectors $(\sin\chi_k, -\cos\chi_k)$ and $(\cos\chi_k, \sin\chi_k)$ rotate in a prograde sense and the vectors obtained from these by the replacement $\chi_k \to -\chi_k$, namely $(-\sin\chi_k, -\cos\chi_k)$ and $(\cos\chi_k, -\sin\chi_k)$ are in retrograde rotation. On resolving the in-phase and out-of-phase vectors in terms of these, one obtains

$$\begin{aligned}
(\Delta\psi^{ip}(t)\sin\epsilon_0,\ \Delta\epsilon^{ip}(t)) &= A_k^{pro\ ip}(\sin\chi_k,\ -\cos\chi_k) + A_k^{ret\ ip}(-\sin\chi_k,\ -\cos\chi_k), \\
(\Delta\psi^{op}(t)\sin\epsilon_0,\ \Delta\epsilon^{op}(t)) &= A_k^{pro\ op}(\cos\chi_k,\ \sin\chi_k) + A_k^{ret\ op}(\cos\chi_k,\ -\sin\chi_k),
\end{aligned} \tag{47}$$

where

$$
\begin{array}{rcl}
A_k^{pro\ ip} & = & \frac{1}{2}\left(q_k \Delta\psi_k^{ip} \sin\epsilon_0 - \Delta\epsilon_k^{ip}\right), \\
A_k^{ret\ ip} & = & -\frac{1}{2}\left(q_k \Delta\psi_k^{ip} \sin\epsilon_0 + \Delta\epsilon_k^{ip}\right), \\
A_k^{pro\ op} & = & \frac{1}{2}\left(\Delta\psi_k^{op} \sin\epsilon_0 + q_k \Delta\epsilon_k^{op}\right), \\
A_k^{ret\ op} & = & \frac{1}{2}\left(\Delta\psi_k^{op} \sin\epsilon_0 - q_k \Delta\epsilon_k^{op}\right).
\end{array}
\tag{48}
$$

The expressions providing the corresponding nutation in longitude and in obliquity from circular terms are

$$
\begin{array}{rcl}
\Delta\psi_k^{ip} & = & \frac{q_k}{\sin\epsilon_0}\left(A_k^{pro\ ip} - A_k^{ret\ ip}\right), \\
\Delta\psi_k^{op} & = & \frac{1}{\sin\epsilon_0}\left(A_k^{pro\ op} + A_k^{ret\ op}\right), \\
\Delta\epsilon_k^{ip} & = & -\left(A_k^{pro\ ip} + A_k^{ret\ ip}\right), \\
\Delta\epsilon_k^{op} & = & q_k\left(A_k^{pro\ op} - A_k^{ret\ op}\right).
\end{array}
\tag{49}
$$

The contribution of the k-term of the nutation to the position of the Celestial Intermediate Pole (CIP) in the mean equatorial frame is thus given by the complex coordinate

$$
\Delta\psi(t)\sin\epsilon_0 + i\Delta\epsilon(t) = -i\left(A_k^{pro} e^{i\chi_k} + A_k^{ret} e^{-i\chi_k}\right), \tag{50}
$$

where A_k^{pro} and A_k^{ret} are the amplitudes of the prograde and retrograde components, respectively, and are given by

$$
A_k^{pro} = A_k^{pro\ ip} + iA_k^{pro\ op}, \qquad A_k^{ret} = A_k^{ret\ ip} + iA_k^{ret\ op}. \tag{51}
$$

The decomposition into prograde and retrograde components is important for studying the role of resonance in nutation because any resonance (especially in the case of the nonrigid Earth) affects A_k^{pro} and A_k^{ret} unequally.

In the literature (Wahr, 1981) one finds an alternative notation, frequently followed in analytic formulations of nutation theory, that is:

$$
\Delta\epsilon(t) + i\Delta\psi(t)\sin\epsilon_0 = -i\left(A_k^{pro\ -} e^{-i\chi_k} + A_k^{ret\ -} e^{i\chi_k}\right), \tag{52}
$$

with

$$
A_k^{pro\ -} = A_k^{pro\ ip} - iA_k^{pro\ op}, \qquad A_k^{ret\ -} = A_k^{ret\ ip} - iA_k^{ret\ op}. \tag{53}
$$

Further detail concerning this topic can be found in Defraigne *et al.*, (1995) and Bizouard *et al.* (1998).

5.10 Procedures and IERS Routines for Transformations from ITRS to GCRS

Fortran routines that implement the IAU 2000 transformations are provided on the IERS Conventions web page, which is at < [2]>.

The following routines are provided:

`BPN2000`	CIO-based intermediate-to-celestial matrix
`CBPN2000`	equinox-based true-to-celestial matrix
`EE2000`	equation of the equinoxes (EE)
`EECT2000`	EE complementary terms
`ERA2000`	Earth Rotation Angle
`GMST2000`	Greenwich Mean Sidereal Time
`GST2000`	Greenwich (apparent) Sidereal Time
`NU2000A`	nutation, IAU 2000A
`NU2000B`	nutation, IAU 2000B
`POM2000`	form polar-motion matrix
`SP2000`	the quantity s'
`T2C2000`	form terrestrial to celestial matrix
`XYS2000A`	X, Y, s

The above routines are to a large extent self-contained, but in some cases use simple utility routines from the IAU *Standards Of Fundamental Astronomy* software collection. This may be found at <[3]>. The SOFA collection includes its own implementations of the IAU 2000 models, together with tools to facilitate their rigorous use. Two equivalent ways to implement the IAU Resolutions in the transformation from ITRS to GCRS provided by expression (1) can be used, namely (a) the new transformation based on the Celestial Intermediate Origin and the Earth Rotation Angle and (b) the classical transformation based on the equinox and Greenwich Sidereal Time. They are called respectively "CIO-based" and "equinox-based" transformations in the following.

For both transformations, the procedure is to form the various components of expression (1), or their classical counterparts, and then to combine these components into the complete terrestrial-to-celestial matrix.

Common to all cases is generating the polar-motion matrix, $W(t)$ in expression (1), by calling `POM2000`. This requires the polar coordinates x_p, y_p and the quantity s'; the latter can be estimated using `SP2000`.

The matrix for the combined effects of nutation, precession and frame bias is $Q(t)$ in expression (1). For the CIO-based transformation, this is the intermediate-to-celestial matrix and can be obtained using the routine `BPN2000`, given the CIP position X, Y and the quantity s that defines the position of the CIO. The IAU 2000A X, Y, s are available by calling the routine `XYS2000A`. In the case of the equinox-based transformation, the counterpart to matrix $Q(t)$ is the true-to-celestial matrix. To obtain this matrix requires the nutation components $\Delta\psi$ and $\Delta\epsilon$; these can be predicted using the IAU 2000A model by means of the routine `NU2000A`. Faster but lower-accuracy predictions are available from the `NU2000B` routine, which implements the IAU 2000B truncated model. Once $\Delta\psi$ and $\Delta\epsilon$ are known, the true-to-celestial matrix can be obtained by calling the routine `CBPN2000`.

The intermediate component is the angle for Earth rotation that defines matrix $R(t)$ in expression (1). For the CIO-based transformation, the angle in question is the Earth Rotation Angle, θ, which can be obtained by calling the routine `ERA2000`. The counterpart in the case of the equinox-based transformation is the Greenwich (apparent) Sidereal Time. This can be obtained by calling the routine `GST2000`, given the nutation in longitude, $\Delta\psi$, that was obtained earlier.

The three components are then assembled into the final terrestrial-to-celestial matrix by means of the routine `T2C2000`.

Three methods of applying the above scheme are set out below.

Method (1): CIO-based transformation consistent with IAU 2000A precession-nutation

This uses the new (X, Y, s, θ) transformation, which is consistent with IAU 2000A precession-nutation. Having called `SP2000` to obtain the quantity s', and knowing the polar motion x_p, y_p, the matrix $W(t)$ can be obtained by calling `POM2000`. The Earth Rotation Angle provided by expression (13) can be predicted with `ERA2000`, as a function of UT1. The X, Y, s series, based on expressions (15) and (16) for X and Y, the coordinates of the CIP, and on Table 5.2c for the quantity s, that defines the position of the CIO, can be generated using the `XYS2000A` routine. (Note that this routine computes the full series for s rather than the summary model in Table 5.2c.) The matrix $Q(t)$ that transforms from the intermediate system to the GCRS coordinates can then be generated by means of `BPN2000`. The finished terrestrial-to-celestial matrix is obtained by calling the `T2C2000` routine, specifying the polar-motion matrix, the Earth Rotation Angle and the intermediate-to-celestial matrix.

Method (2A): the equinox-based transformation, using IAU 2000A precession-nutation

An alternative is the classical, equinox-based, transformation, using the IAU 2000A precession-nutation model and the new IAU-2000-compatible expression for GST.

[3] http://www.iau-sofa.rl.ac.uk

As for Method 1, the first step is to use SP2000 and POM2000 to obtain the matrix $W(t)$, given x_p, y_p. Next, compute the nutation components (lunisolar + planetary) by calling NU2000A. The Greenwich (apparent) Sidereal Time is predicted by calling GST2000. This requires $\Delta\psi$ and TT as well as UT1. The matrix that transforms from the true equator and equinox of date to GCRS coordinates can then be generated by means of CBPN2000. Finally, the finished terrestrial-to-celestial matrix is obtained by calling the T2C2000 routine, specifying the polar-motion matrix, the Greenwich Sidereal Time and the intermediate-to-celestial matrix.

Method (2B): the classical transformation, using IAU 2000B precession-nutation

The third possibility is to carry out the classical transformation as for Method 2A, but based on the truncated IAU 2000B precession-nutation model. Using IAU 2000B limits the accuracy to about 1 mas, but the computations are significantly less onerous than when using the full IAU 2000A model.

The same procedure as in Method (2A) is used, but substituting NU2000B for NU2000A. Depending on the accuracy requirements, further efficiency optimizations are possible, including setting s' to zero, omitting the equation of the equinoxes complementary terms and even neglecting the polar motion.

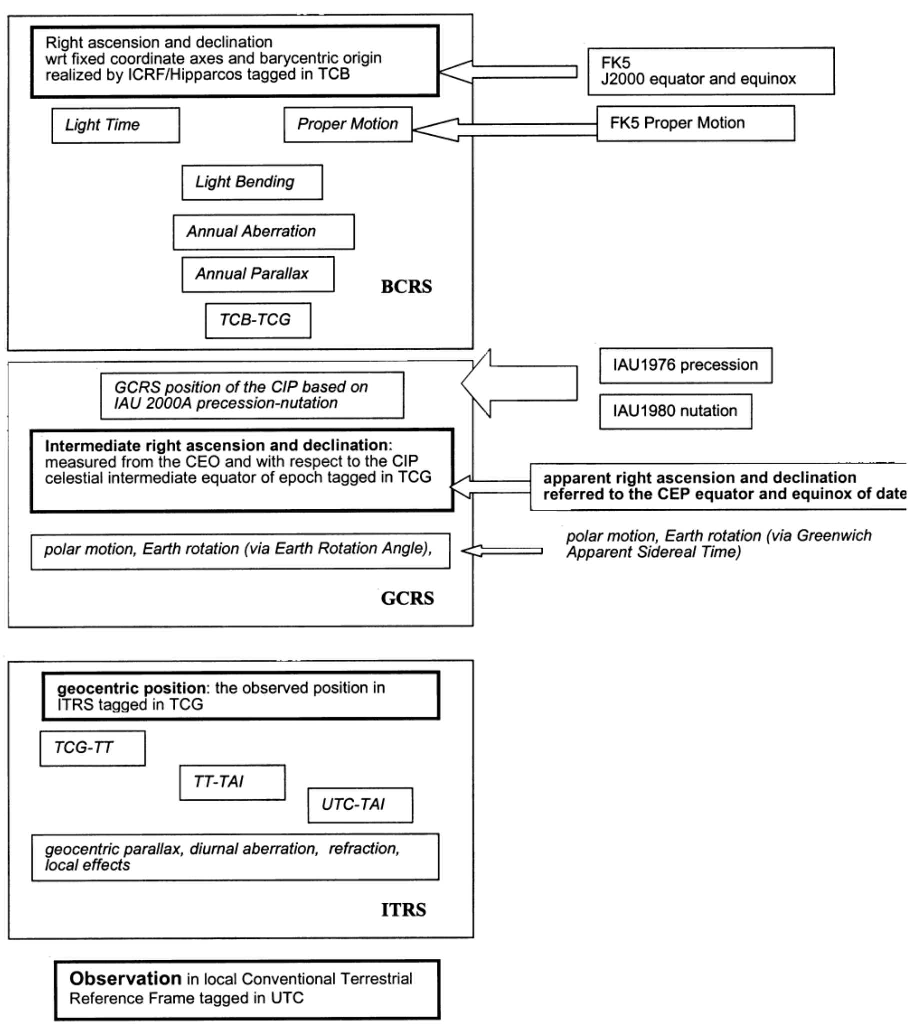

Figure 1. Process to transform from celestial to terrestrial systems. Differences with the past process are shown on the right of the diagram.

Fig. 5.1 Process to transform from celestial to terrestrial systems. Differences with the past process are shown on the right of the diagram.

5.11 Notes on the new Procedure to Transform from ICRS to ITRS

The transformation from the GCRS to ITRS, which is provided in detail in this chapter for use in the IERS Conventions, is also part of the more general transformation for computing directions of celestial objects in intermediate systems.

The procedure to be followed in transforming from the celestial (ICRS) to the terrestrial (ITRS) systems has been clarified to be consistent with the improving observational accuracy. See Figure 5.1 (McCarthy and Capitaine (in Capitaine *et al.*, 2002)) for a diagram of the new and old procedures to be followed. As before, we make use of an intermediate reference system in transforming to a terrestrial system. In this case we call that system the Intermediate Celestial Reference System. (See also Seidelmann and Kovalevsky (2002).)

The Celestial Intermediate Pole (CIP) that is realized by the IAU2000A/B precession-nutation model defines its equator and the Conventional Intermediate Origin replaces the equinox.

The position in this reference system is called the intermediate right ascension and declination and is analogous to the previous designation of "apparent right ascension and declination."

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