6 Geopotential (22 March 2006)

Gravitational models commonly used in current (2003) precision orbital analysis by contributors to the International Laser Ranging Service (ILRS) include EGM96 (Lemoine *et al.*, 1998), JGM-3 (Tapley *et al.*, 1996), and GRIM5-C1 (Gruber *et al.*, 2000). For products of interest to IERS, similar accuracy is achievable with any of these models. IERS, recognizing the continuous development of new gravitational models, and anticipating the results of upcoming geopotential mapping missions, recommends at this time the EGM96 model as the conventional model. The $GM_\oplus$ and a_e values reported with EGM96 (398600.4415 km^3/s^2 and 6378136.3 m) should be used as scale parameters with the geopotential coefficients. The recommended $GM_\oplus = 398600.4418$ should be used with the two-body term when working with Geocentric Coordinate Time (TCG) (398600.4415 or 398600.4356 should be used by those still working with Terrestrial Time (TT) or Barycentric Dynamical Time (TDB) units, respectively). EGM96 is available at <[1]>.

If the geopotential is expanded about an origin at the Earth's center of mass, then the degree one geopotential harmonic coefficients vanish and the spherical harmonic expansion begins at degree two. If the geopotential is expanded in a crust-fixed frame (which is offset from the mass center by the geocenter motion) through the inclusion of degree one harmonics, then Coriolis-type accelerations must be included to account for the diurnal motion of the crust-fixed frame about the mass center; this approach is not recommended and is not considered further here.

Values for the C_{21} and S_{21} coefficients are included in the EGM96 model. The C_{21} and S_{21} coefficients describe the position of the Earth's figure axis. When averaged over many years, the figure axis should closely coincide with the observed position of the rotation pole averaged over the same time period. Any differences between the mean figure and mean rotation pole averaged would be due to long-period fluid motions in the atmosphere, oceans, or Earth's fluid core (Wahr, 1987; 1990). At present, there is no independent evidence that such motions are important. The EGM96 values for C_{21} and S_{21} give a mean figure axis that corresponds to the mean pole position recommended in Chapter 4 Terrestrial Reference Frame.

This choice for C_{21} and S_{21} is realized as follows. First, to use the geopotential coefficients to solve for a satellite orbit, it is necessary to rotate from the Earth-fixed frame, where the coefficients are pertinent, to an inertial frame, where the satellite motion is computed. This transformation between frames should include polar motion. We assume the polar motion parameters used are relative to the IERS Reference Pole. If $\bar{x}$ and $\bar{y}$ are the angular displacements of the pole of the Terrestrial Reference Frame described in Chapter 4 relative to the IERS Reference Pole, then the values

$$
\begin{aligned}
\bar{C}_{21} &= \sqrt{3}\bar{x}\bar{C}_{20} - \bar{x}\bar{C}_{22} + \bar{y}\bar{S}_{22}, \\
\bar{S}_{21} &= -\sqrt{3}\bar{y}\bar{C}_{20} - \bar{y}\bar{C}_{22} - \bar{x}\bar{S}_{22},
\end{aligned}
$$

where $\bar{x} = 0.262 \times 10^{-6}$ radians (equivalent to 0.054 arcsec) and $\bar{y} = 1.730 \times 10^{-6}$ radians (equivalent to 0.357 arcsec) are those derived by the IERS Earth Orientation Centre (see subsection 7.1.4), so that the mean figure axis coincides with the pole described in Chapter 4. The EGM96 values at 1 January 2000 are $\bar{C}_{20} = -4.84165209 \times 10^{-4}$ (tide free), and $d\bar{C}_{20}/dt = +1.162755 \times 10^{-11}$/year.

This gives normalized coefficients of

$$
\begin{aligned}
\bar{C}_{21}(\text{IERS}) &= -2.23 \times 10^{-10}, \quad \text{and} \\
\bar{S}_{21}(\text{IERS}) &= 14.48 \times 10^{-10}.
\end{aligned}
$$

$\bar{C}_{21}$ and $\bar{S}_{21}$ are time variable. The values above are associated with the epoch of 1 January 2000. The complete definition of the instantaneous values of the two coefficients to use when computing orbits is given by:

$$
\begin{aligned}
\bar{C}_{21} &= \bar{C}_{21}(t_0) + d\bar{C}_{21}/dt[t - t_0], \quad \text{and} \\
\bar{S}_{21} &= \bar{S}_{21}(t_0) + d\bar{S}_{21}/dt[t - t_0],
\end{aligned}
$$

[1] http://www.nima.mil/GandG/wgsegm/egm96.html

where $d\bar{C}_{21}/dt$ and $d\bar{S}_{21}/dt$ are the time derivatives determined at epoch t_0 to be -0.337×10^{-11}/y and $+1.606 \times 10^{-11}$/y respectively. It is also necessary to account for the solid Earth and ocean pole tide described later in this chapter.

6.1 Effect of Solid Earth Tides

The changes induced by the solid Earth tides in the free space potential are most conveniently modeled as variations in the standard geopotential coefficients C_{nm} and S_{nm} (Eanes *et al.*, 1983). The contributions ΔC_{nm} and ΔS_{nm} from the tides are expressible in terms of the k Love number. The effects of ellipticity and of the Coriolis force due to Earth rotation on tidal deformations necessitates the use of three k parameters, $k_{nm}^{(0)}$ and $k_{nm}^{(\pm)}$ (except for $n = 2$) to characterize the changes produced in the free space potential by tides of spherical harmonic degree and order (nm) (Wahr, 1981); only two parameters are needed for $n = 2$ because $k_{2m}^{(-)} = 0$ is zero due to mass conservation.

Anelasticity of the mantle causes $k_{nm}^{(0)}$ and $k_{nm}^{(\pm)}$ to acquire small imaginary parts (reflecting a phase lag in the deformational response of the Earth to tidal forces), and also gives rise to a variation with frequency which is particularly pronounced within the long period band. Though modeling of anelasticity at the periods relevant to tidal phenomena (8 hours to 18.6 years) is not yet definitive, it is clear that the magnitudes of the contributions from anelasticity cannot be ignored (see below). Recent evidence relating to the role of anelasticity in the accurate modeling of nutation data (Mathews *et al.*, 2002) lends support to the model employed herein, at least up to diurnal tidal periods; and there is no compelling reason at present to adopt a different model for the long period tides.

Solid Earth tides within the diurnal tidal band (for which $(nm) = (21)$) are not wholly due to the direct action of the tide generating potential (TGP) on the solid Earth; they include the deformations (and associated geopotential changes) arising from other effects of the TGP, namely, ocean tides and wobbles of the mantle and the core regions. Deformation due to wobbles arises from the incremental centrifugal potentials caused by the wobbles; and ocean tides load the crust and thus cause deformations. Anelasticity affects the Earth's deformational response to all these types of forcing.

The wobbles, in turn, are affected by changes in the Earth's moment of inertia due to deformations from all sources, and in particular, from the deformation due to loading by the $(nm) = (21)$ part of the ocean tide; wobbles are also affected by the anelasticity contributions to all deformations, and by the coupling of the fluid core to the mantle and the inner core through the action of magnetic fields at its boundaries (Mathews *et al.*, 2002). Resonances in the wobbles—principally, the Nearly Diurnal Free Wobble resonance associated with the Free Core Nutation (FCN)—and the consequent resonances in the contribution to tidal deformation from the centrifugal perturbations associated with the wobbles, cause the body tide and load Love/Shida number parameters of the diurnal tides to become strongly frequency dependent. For the derivation of resonance formulae of the form (6) below to represent this frequency dependence, see Mathews *et al.*, (1995). The resonance expansions assume that the Earth parameters entering the wobble equations are all frequency independent. However the ocean tide induced deformation makes a frequency dependent contribution to deformability parameters which are among the Earth parameters just referred to. It becomes necessary therefore to add small corrections to the Love number parameters computed using the resonance formulae. These corrections are included in the tables of Love number parameters given in this chapter and the next.

The deformation due to ocean loading is itself computed in the first place using frequency independent load Love numbers (see the sections 6.5 and 7.1). Corrections to take account of the resonances in the load Love numbers are incorporated through equivalent corrections to the *body tide* Love numbers, following Wahr and Sasao (1981), as explained further below. These corrections are also included in the tables of Love numbers.

The degree 2 tides produce time dependent changes in C_{2m} and S_{2m}, through $k_{2m}^{(0)}$, which can exceed 10^{-8} in magnitude. They also produce changes exceeding 3×10^{-12} in C_{4m} and S_{4m} through $k_{2m}^{(+)}$. (The direct contributions of the degree 4 tidal potential to these coefficients are negligible.) The only other changes exceeding this cutoff are in C_{3m} and S_{3m}, produced by the degree 3 part of the tide generating potential.

The computation of the tidal contributions to the geopotential coefficients is most efficiently done by a three-step procedure. In Step 1, the $(2m)$ part of the tidal potential is evaluated in the time domain for each m using lunar and solar ephemerides, and the corresponding changes ΔC_{2m} and ΔS_{2m} are computed using frequency independent nominal values k_{2m} for the respective $k_{2m}^{(0)}$. The contributions of the degree 3 tides to C_{3m} and S_{3m} through $k_{3m}^{(0)}$ and also those of the degree 2 tides to C_{4m} and S_{4m} through $k_{2m}^{(+)}$ may be computed by a similar procedure; they are at the level of 10^{-11}.

Step 2 corrects for the deviations of the $k_{21}^{(0)}$ of several of the constituent tides of the diurnal band from the constant nominal value k_{21} assumed for this band in the first step. Similar corrections need to be applied to a few of the constituents of the other two bands also.

Steps 1 and 2 can be used to compute the total tidal contribution, including the time independent (permanent) contribution to the geopotential coefficient $\bar{C}_{20}$, which is adequate for a "conventional tide free" model such as EGM96. When using a "zero tide" model, this permanent part should not be counted twice, this is the goal of Step 3 of the computation. See section 6.4.

With frequency-independent values k_{nm} (Step 1), changes induced by the (nm) part of the tide generating potential in the normalized geopotential coefficients having the same (nm) are given in the time domain by

$$\Delta\bar{C}_{nm} - i\Delta\bar{S}_{nm} = \frac{k_{nm}}{2n+1}\sum_{j=2}^{3}\frac{GM_j}{GM_\oplus}\left(\frac{R_e}{r_j}\right)^{n+1}\bar{P}_{nm}(\sin\Phi_j)e^{-im\lambda_j} \tag{1}$$

(with $\bar{S}_{n0} = 0$), where

k_{nm} = nominal Love number for degree n and order m,

R_e = equatorial radius of the Earth,

$GM_\oplus$ = gravitational parameter for the Earth,

GM_j = gravitational parameter for the Moon ($j = 2$) and Sun ($j = 3$),

r_j = distance from geocenter to Moon or Sun,

Φ_j = body fixed geocentric latitude of Moon or Sun,

λ_j = body fixed east longitude (from Greenwich) of Moon or Sun,

and $\bar{P}_{nm}$ is the normalized associated Legendre function related to the classical (unnormalized) one by

$$\bar{P}_{nm} = N_{nm}P_{nm}, \tag{2a}$$

where

$$N_{nm} = \sqrt{\frac{(n-m)!(2n+1)(2-\delta_{om})}{(n+m)!}}. \tag{2b}$$

Correspondingly, the normalized geopotential coefficients $(\bar{C}_{nm}, \bar{S}_{nm})$ are related to the unnormalized coefficients (C_{nm}, S_{nm}) by

$$C_{nm} = N_{nm}\bar{C}_{nm}, \quad S_{nm} = N_{nm}\bar{S}_{nm}. \tag{3}$$

Equation (1) yields $\Delta\bar{C}_{nm}$ and $\Delta\bar{S}_{nm}$ for both $n = 2$ and $n = 3$ for all m, apart from the corrections for frequency dependence to be evaluated in Step 2. (The particular case $(nm) = (20)$ needs special consideration, however, as already indicated.)

One further computation to be done in Step 1 is that of the changes in the degree 4 coefficients produced by the degree 2 tides. They are given by

$$\Delta\bar{C}_{4m} - i\Delta\bar{S}_{4m} \quad = \quad \frac{k_{2m}^{(+)}}{5} \sum_{j=2}^{3} \frac{GM_j}{GM_\oplus} \left(\frac{R_e}{r_j}\right)^3 \bar{P}_{2m}(\sin\Phi_j) e^{-im\lambda_j}, \quad (m = 0, 1, 2), \quad (4)$$

which has the same form as Equation (1) for $n = 2$ except for the replacement of k_{2m} by $k_{2m}^{(+)}$.

The parameter values for the computations of Step 1 are given in Table 6.1. The choice of these nominal values has been made so as to minimize the number of terms for which corrections will have to be applied in Step 2. The nominal value for $m = 0$ has to be chosen real because there is no closed expression for the contribution to $\bar{C}_{20}$ from the imaginary part of $k_{20}^{(0)}$.

Table 6.1 Nominal values of solid Earth tide external potential Love numbers.

		Elastic Earth		Anelastic Earth		
n	m	k_{nm}	k_{nm}^{+}	Re k_{nm}	Im k_{nm}	k_{nm}^{+}
2	0	0.29525	−0.00087	0.30190	−0.00000	−0.00089
2	1	0.29470	−0.00079	0.29830	−0.00144	−0.00080
2	2	0.29801	−0.00057	0.30102	−0.00130	−0.00057
3	0	0.093	···			
3	1	0.093	···			
3	2	0.093	···			
3	3	0.094	···			

The frequency dependence corrections to the $\Delta\bar{C}_{nm}$ and $\Delta\bar{S}_{nm}$ values obtained from Step 1 are computed in Step 2 as the sum of contributions from a number of tidal constituents belonging to the respective bands. The contribution to $\Delta\bar{C}_{20}$ from the long period tidal constituents of various frequencies f is

$$\mathrm{Re}\sum_{f(2,0)}(A_0\delta k_f H_f)\, e^{i\theta_f} \quad = \quad \sum_{f(2,0)}[(A_0 H_f \delta k_f^R)\cos\theta_f - (A_0 H_f \delta k_f^I)\sin\theta_f)], \qquad (5a)$$

while the contribution to $(\Delta\bar{C}_{21} - i\Delta\bar{S}_{21})$ from the diurnal tidal constituents and to $\Delta\bar{C}_{22} - i\Delta\bar{S}_{22}$ from the semidiurnals are given by

$$\Delta\bar{C}_{2m} - i\Delta\bar{S}_{2m} = \eta_m \sum_{f(2,m)} (A_m\delta k_f H_f)\, e^{i\theta_f}, \quad (m = 1, 2), \qquad (5b)$$

where

$$A_0 = \frac{1}{R_e\sqrt{4\pi}} = 4.4228 \times 10^{-8}\ \mathrm{m}^{-1}, \qquad (5c)$$

$$A_m = \frac{(-1)^m}{R_e\sqrt{8\pi}} = (-1)^m(3.1274 \times 10^{-8})\ \mathrm{m}^{-1}, \qquad (m \neq 0), \qquad (5d)$$

$$\eta_1 = -i,\ \eta_2 = 1, \qquad (5e)$$

δk_f = difference between k_f defined as $k_{2m}^{(0)}$ at frequency f and the nominal value k_{2m}, in the sense $k_f - k_{2m}$, plus a contribution from ocean loading,

δk_f^R = real part of δk_f, and

δk_f^I = imaginary part of δk_f, i.e., $\delta k_f = \delta k_f^R + i\delta k_f^I$,

H_f = amplitude (in meters) of the term at frequency f from the harmonic expansion of the tide generating potential, defined according to the convention of Cartwright and Tayler (1971), and

θ_f = $\bar{n} \cdot \bar{\beta} = \sum_{i=1}^{6} n_i \beta_i$, or

θ_f = $m(\theta_g + \pi) - \bar{N} \cdot \bar{F} = m(\theta_g + \pi) - \sum_{j=1}^{5} N_j F_j$,

where

$\bar{\beta}$ = six-vector of Doodson's fundamental arguments β_i, (τ, s, h, p, N', p_s),

$\bar{n}$ = six-vector of multipliers n_i (for the term at frequency f) of the fundamental arguments,

$\bar{F}$ = five-vector of fundamental arguments F_j (the Delaunay variables l, l', F, D, Ω) of nutation theory,

$\bar{N}$ = five-vector of multipliers N_i of the Delaunay variables for the nutation of frequency $-f + d\theta_g/dt$,

and θ_g is the Greenwich Mean Sidereal Time expressed in angle units (*i.e.* $24^h = 360°$; see Chapter 5).

(π in $(\theta_g + \pi)$ is now to be replaced by 180.)

For the fundamental arguments (l, l', F, D, Ω) of nutation theory and the convention followed here in choosing their multipliers N_j, see Chapter 5. For conversion of tidal amplitudes defined according to different conventions to the amplitude H_f corresponding to the Cartwright-Tayler convention, use Table 6.5 given at the end of this chapter.

For diurnal tides, the frequency dependent values of any load or body tide Love number parameter L (such as $k_{21}^{(0)}$ or $k_{21}^{(+)}$ in the present context) may be represented as a function of the tidal excitation frequency σ by a resonance formula

$$L(\sigma) = L_0 + \sum_{\alpha=1}^{3} \frac{L_\alpha}{(\sigma - \sigma_\alpha)}, \tag{6}$$

except for the small corrections referred to earlier. (They are to take account of frequency dependent contributions to a few of the Earth's deformability parameters, which make (6) inexact.) The σ_α, $(\alpha = 1, 2, 3)$, are the respective resonance frequencies associated with the Chandler wobble (CW), the retrograde free core nutation (FCN), and the prograde free core nutation (PFCN, also known as the free inner core nutation, FICN), and the L_α are the corresponding resonance coefficients. All the parameters are complex. The σ_α and σ are expressed in cycles per sidereal day, with the convention that positive (negative) frequencies represent retrograde (prograde) waves. (This sign convention, followed in tidal theory, is the opposite of that employed in analytical theories of nutation.) In particular, given the tidal frequency f in degrees per hour, one has

$$\sigma = f/(15 \times 1.002737909),$$

the factor 1.002737909 being the number of sidereal days per solar day. The values used herein for the σ_α are from Mathews *et al.* (2002), adapted to the sign convention used here:

$$\begin{array}{lclcl} \sigma_1 & = & -0.0026010 & - & 0.0001361\, i \\ \sigma_2 & = & 1.0023181 & + & 0.000025\, i \\ \sigma_3 & = & 0.999026 & + & 0.000780\, i. \end{array} \tag{7}$$

They were estimated from a fit of nutation theory to precession rate and nutation amplitude estimates found from an analyis of very long baseline interferometry (VLBI) data.

Table 6.2 lists the values of L_0 and L_α in resonance formulae of the form (6) for $k_{21}^{(0)}$ and $k_{21}^{(+)}$. They were obtained by evaluating the relevant expressions from Mathews *et al.* (1995), using values taken from computations of Buffett and Mathews (unpublished) for the needed deformability parameters together with values obtained for the wobble resonance parameters in the course of computations of the nutation results of Mathews *et al.* (2002). The deformability parameters for an elliptical, rotating, elastic, and oceanless Earth model based on the 1 sec PREM with the ocean layer replaced by solid, and corrections to these for the effects of mantle anelasticity, were found by integration of the tidal deformation equations. Anelasticity computations were based on the Widmer *et al.* (1991) model of mantle Q. As in Wahr and Bergen (1986), a power law was assumed for the frequency dependence of Q, with 200 s as the reference period; the value $\alpha = 0.15$ was used for the power law index. The anelasticity contribution (out-of-phase and in-phase) to the tidal changes in the geopotential coefficients is at the level of one to two percent in-phase, and half to one percent out-of-phase, *i.e.*, of the order of 10^{-10}. The effects of anelasticity, ocean loading and currents, and electromagnetic couplings on the wobbles result in indirect contributions to $k_{21}^{(0)}$ and $k_{21}^{(+)}$ which are almost fully accounted for through the values of the wobble resonance parameters. Also shown in Table 6.2 are the resonance parameters for the load Love numbers h'_{21}, k'_{21}, and l'_{21}, which are relevant to the solid Earth deformation caused by ocean tidal loading and to the consequential changes in the geopotential. (Only the real parts are shown: the small imaginary parts make no difference to the effect to be now considered which is itself small.)

Table 6.2 Parameters in the resonance formulae for $k_{21}^{(0)}$, $k_{21}^{(+)}$ and the load Love numbers.

	$k^{(0)}$		$k^{(+)}$	
α	Re L_α	Im L_α	Re L_α	Im L_α
0	0.29954	-0.1412×10^{-2}	-0.804×10^{-3}	0.237×10^{-5}
1	-0.77896×10^{-3}	-0.3711×10^{-4}	0.209×10^{-5}	0.103×10^{-6}
2	0.90963×10^{-4}	-0.2963×10^{-5}	-0.182×10^{-6}	0.650×10^{-8}
3	-0.11416×10^{-5}	0.5325×10^{-7}	-0.713×10^{-9}	-0.330×10^{-9}
Load Love Numbers (Real parts only)				
	h'_{21}	l'_{21}	k'_{21}	
0	-0.99500	0.02315	-0.30808	
1	1.6583×10^{-3}	2.3232×10^{-4}	8.1874×10^{-4}	
2	2.8018×10^{-4}	-8.4659×10^{-6}	1.4116×10^{-4}	
3	5.5852×10^{-7}	1.0724×10^{-8}	3.4618×10^{-7}	

The expressions given in section 6.5 for the contributions from ocean tidal loading assume the constant nominal value $k_2'^{(nom)} = -0.3075$ for k' of the degree 2 tides. Further contributions arise from the frequency dependence of k'_{21}. These may be expressed, following Wahr and Sasao (1981), in terms of an effective ocean tide contribution $\delta k^{(OT)}(\sigma)$ to the body tide Love number $k_{21}^{(0)}$:

$$\delta k^{(OT)}(\sigma) = [k'_{21}(\sigma) - k_2'^{(nom)}] \left(\frac{4\pi G\rho_w R}{5\bar{g}}\right) A_{21}(\sigma), \tag{8}$$

where G is the constant of universal gravitation, ρ_w is the density of sea water (1025 kg m^{-3}), R is the Earth's mean radius (6.371×10^6 m), $\bar{g}$ is the mean acceleration due to gravity at the Earth's surface (9.820 m s^{-2}), and $A_{21}(\sigma)$ is the admittance for the degree 2 tesseral component of the ocean tide of frequency σ cpsd:

$$A_{21}(\sigma) = \zeta_{21}(\sigma)/\bar{H}(\sigma).$$

ζ_{21} is the complex amplitude of the height of the $(nm) = (21)$ component of the ocean tide, and $\bar{H}$ is the height equivalent of the amplitude of the tide generating potential, the bar being a reminder that the spherical harmonics used in defining the two amplitudes should be identically normalized. Wahr and Sasao (1981) employed the factorized form

$$A_{21}(\sigma) = f_{FCN}(\sigma)\, f_{OD}(\sigma),$$

wherein the first factor represents the effect of the FCN resonance, and the second, that of other ocean dynamic factors. The following empirical formulae (Mathews *et al.*, 2002) which provide good fits to the FCN factors of a set of 11 diurnal tides (Desai and Wahr, 1995) and to the admittances obtainable from the ocean load angular momenta of four principal tides (Chao *et al.*, 1996) are used herein:

$$f_{OD}(\sigma) = (1.3101 - 0.8098\, i) - (1.1212 - 0.6030\, i)\sigma,$$
$$f_{FCN}(\sigma) = 0.1732 + 0.9687\, f_{eqm}(\sigma),$$
$$f_{eqm}(\sigma) = \frac{\gamma(\sigma)}{1 - (3\rho_w/5\bar{\rho})\gamma'(\sigma)},$$

where $\gamma = 1 + k - h$ and $\gamma' = 1 + k' - h'$, $\bar{\rho}$ is the Earth's mean density. (Here k stands for $k_{21}^{(0)}$, and similarly for the other symbols. Only the real parts need be used.) f_{eqm} is the FCN factor for a global equilibrium ocean.

Table 6.3a shows the values of

$$\delta k_f \equiv (k_{21}^{(0)}(\sigma) - k_{21}) + \delta k_{21}^{OT}(\sigma),$$

along with the real and imaginary parts of the amplitude $(A_1 \delta k_f H_f)$. The tides listed are those for which either of the parts is at least 10^{-13} after round-off. (A cutoff at this level is used for the individual terms in order that accuracy at the level of 3×10^{-12} be not affected by the accumulated contributions from the numerous smaller terms that are disregarded.) Roughly half the value of the imaginary part comes from the ocean tide term, and the real part contribution from this term is of about the same magnitude.

The values used for $k_{21}^{(0)}(\sigma)$ in evaluating δk_f are from an exact computation necessarily involving use of the framework of nutation-wobble theory which is outside the scope of this chapter. If the (approximate) resonance formula were used instead for the computation, the resulting numbers for δk_f^R and δk_f^I would require small corrections to match the exact values. In units of 10^{-5}, they are (in-phase, out-of-phase) (1, 1) for Q_1, (1, 1) for O_1 and its companion having Doodson numbers 145,545, (1, 0) for NO_1, $(0, -1)$ for P_1, (244, 299) for ψ_1, (12, 12) for ϕ_1, (3, 2) for J_1, and (2, 1) for OO_1 and its companion with Doodson numbers 185,565. These are the only tides for which the corrections would contribute nonneglibily to the numbers listed in the last two columns of the table.

Calculation of the correction due to any tidal constituent is illustrated by the following example for K_1. Given that $A_m = A_1 = -3.1274 \times 10^{-8}$, and that $H_f = 0.36870$, $\theta_f = (\theta_g + \pi)$, and $k_{21}^{(0)} = (0.25746 + 0.00118\, i)$ for this tide, one finds on subtracting the nominal value $(0.29830 - 0.00144\, i)$ that $\delta k_f = (-0.04084 + 0.00262\, i)$. Equation (5b) then yields:

$$\begin{aligned}(\Delta\bar{C}_{21})_{K_1} &= 470.9 \times 10^{-12} \sin(\theta_g + \pi) - 30.2 \times 10^{-12} \cos(\theta_g + \pi),\\ (\Delta\bar{S}_{21})_{K_1} &= 470.9 \times 10^{-12} \cos(\theta_g + \pi) + 30.2 \times 10^{-12} \sin(\theta_g + \pi).\end{aligned}$$

The variation of $k_{20}^{(0)}$ across the zonal tidal band, $(nm) = (20)$, is due to mantle anelasticity; it is described by the formula

$$k_{20}^{(0)} = 0.29525 - 5.796 \times 10^{-4} \left\{ \cot\frac{\alpha\pi}{2} \left[1 - \left(\frac{f_m}{f}\right)^{\alpha} \right] + i \left(\frac{f_m}{f}\right)^{\alpha} \right\} \qquad (9)$$

Table 6.3a The in-phase (ip) amplitudes ($A_1 \delta k_f^R H_f$) and the out-of-phase (op) amplitudes ($A_1 \delta k_f^I H_f$) of the corrections for frequency dependence of $k_{21}^{(0)}$, taking the nominal value k_{21} for the diurnal tides as $(0.29830 - i\,0.00144)$. Units: 10^{-12}. The entries for δk_f^R and δk_f^I are in units of 10^{-5}. Multipliers of the Doodson arguments identifying the tidal terms are given, as also those of the Delaunay variables characterizing the nutations produced by these terms.

Name	deg/hr	Doodson No.	τ	s	h	p	N'	p_s	ℓ	ℓ'	F	D	Ω	δk_f^R 10^{-5}	δk_f^I 10^{-5}	Amp. (ip)	Amp. (op)
$2Q_1$	12.85429	125,755	1	-3	0	2	0	0	2	0	2	0	2	-29	3	-0.1	0.0
σ_1	12.92714	127,555	1	-3	2	0	0	0	0	0	2	2	2	-30	3	-0.1	0.0
	13.39645	135,645	1	-2	0	1	-1	0	1	0	2	0	1	-45	5	-0.1	0.0
Q_1	13.39866	135,655	1	-2	0	1	0	0	1	0	2	0	2	-46	5	-0.7	0.1
ρ_1	13.47151	137,455	1	-2	2	-1	0	0	-1	0	2	2	2	-49	5	-0.1	0.0
	13.94083	145,545	1	-1	0	0	-1	0	0	0	2	0	1	-82	7	-1.3	0.1
O_1	13.94303	145,555	1	-1	0	0	0	0	0	0	2	0	2	-83	7	-6.8	0.6
τ_1	14.02517	147,555	1	-1	2	0	0	0	0	0	0	2	0	-91	9	0.1	0.0
$N\tau_1$	14.41456	153,655	1	0	-2	1	0	0	1	0	2	-2	2	-168	14	0.1	0.0
	14.48520	155,445	1	0	0	-1	-1	0	-1	0	2	0	1	-193	16	0.1	0.0
LK_1	14.48741	155,455	1	0	0	-1	0	0	-1	0	2	0	2	-194	16	0.4	0.0
NO_1	14.49669	155,655	1	0	0	1	0	0	1	0	0	0	0	-197	16	1.3	-0.1
	14.49890	155,665	1	0	0	1	1	0	1	0	0	0	1	-198	16	0.3	0.0
χ_1	14.56955	157,455	1	0	2	-1	0	0	-1	0	0	2	0	-231	18	0.3	0.0
	14.57176	157,465	1	0	2	-1	1	0	-1	0	0	2	1	-233	18	0.1	0.0
π_1	14.91787	162,556	1	1	-3	0	0	1	0	1	2	-2	2	-834	58	-1.9	0.1
	14.95673	163,545	1	1	-2	0	-1	0	0	0	2	-2	1	-1117	76	0.5	0.0
P_1	14.95893	163,555	1	1	-2	0	0	0	0	0	2	-2	2	-1138	77	-43.4	2.9
	15.00000	164,554	1	1	-1	0	0	-1	0	-1	2	-2	2	-1764	104	0.6	0.0
S_1	15.00000	164,556	1	1	-1	0	0	1	0	1	0	0	0	-1764	104	1.6	-0.1
	15.02958	165,345	1	1	0	-2	-1	0	-2	0	2	0	1	-3048	92	0.1	0.0
	15.03665	165,535	1	1	0	0	-2	0	0	0	0	0	-2	-3630	195	0.1	0.0
	15.03886	165,545	1	1	0	0	-1	0	0	0	0	0	-1	-3845	229	-8.8	0.5
K_1	15.04107	165,555	1	1	0	0	0	0	0	0	0	0	0	-4084	262	470.9	-30.2
	15.04328	165,565	1	1	0	0	1	0	0	0	0	0	1	-4355	297	68.1	-4.6
	15.04548	165,575	1	1	0	0	2	0	0	0	0	0	2	-4665	334	-1.6	0.1
	15.07749	166,455	1	1	1	-1	0	0	-1	0	0	1	0	85693	21013	0.1	0.0
	15.07993	166,544	1	1	1	0	-1	-1	0	-1	0	0	-1	35203	2084	-0.1	0.0
ψ_1	15.08214	166,554	1	1	1	0	0	-1	0	-1	0	0	0	22794	358	-20.6	-0.3
	15.08214	166,556	1	1	1	0	0	1	0	1	-2	2	-2	22780	358	0.3	0.0
	15.08434	166,564	1	1	1	0	1	-1	0	-1	0	0	1	16842	-85	-0.3	0.0
	15.11392	167,355	1	1	2	-2	0	0	-2	0	0	2	0	3755	-189	-0.2	0.0
	15.11613	167,365	1	1	2	-2	1	0	-2	0	0	2	1	3552	-182	-0.1	0.0
ϕ_1	15.12321	167,555	1	1	2	0	0	0	0	0	-2	2	-2	3025	-160	-5.0	0.3
	15.12542	167,565	1	1	2	0	1	0	0	0	-2	2	-1	2892	-154	0.2	0.0
	15.16427	168,554	1	1	3	0	0	-1	0	-1	-2	2	-2	1638	-93	-0.2	0.0
θ_1	15.51259	173,655	1	2	-2	1	0	0	1	0	0	-2	0	370	-20	-0.5	0.0
	15.51480	173,665	1	2	-2	1	1	0	1	0	0	-2	1	369	-20	-0.1	0.0
	15.58323	175,445	1	2	0	-1	-1	0	-1	0	0	0	-1	325	-17	0.1	0.0
J_1	15.58545	175,455	1	2	0	-1	0	0	-1	0	0	0	0	324	-17	-2.1	0.1
	15.58765	175,465	1	2	0	-1	1	0	-1	0	0	0	1	323	-16	-0.4	0.0
SO_1	16.05697	183,555	1	3	-2	0	0	0	0	0	0	-2	0	194	-8	-0.2	0.0
	16.12989	185,355	1	3	0	-2	0	0	-2	0	0	0	0	185	-7	-0.1	0.0
OO_1	16.13911	185,555	1	3	0	0	0	0	0	0	-2	0	-2	184	-7	-0.6	0.0
	16.14131	185,565	1	3	0	0	1	0	0	0	-2	0	-1	184	-7	-0.4	0.0
	16.14352	185,575	1	3	0	0	2	0	0	0	-2	0	0	184	-7	-0.1	0.0
ν_1	16.68348	195,455	1	4	0	-1	0	0	-1	0	-2	0	-2	141	-4	-0.1	0.0
	16.68569	195,465	1	4	0	-1	1	0	-1	0	-2	0	-1	141	-4	-0.1	0.0

on the basis of the anelasticity model referred to earlier. Here f is the frequency of the zonal tidal constituent, f_m is the reference frequency equivalent to a period of 200 s, and $\alpha = 0.15$. The δk_f in Table 6.3b are the differences between $k_{20}^{(0)}$ computed from the above formula and the nominal value $k_{20} = 0.30190$ given in Table 6.1.

The total variation in geopotential coefficient $\bar{C}_{20}$ is obtained by adding to the result of Step 1 the sum of the contributions from the tidal constituents listed in Table 6.3b computed using equation (5a). The tidal variations in $\bar{C}_{2m}$ and $\bar{S}_{2m}$ for the other m are computed similarly, except that equation (5b) is to be used together with Table 6.3a for $m = 1$ and Table 6.3c for $m = 2$.

Table 6.3b Corrections for frequency dependence of $k_{20}^{(0)}$ of the zonal tides due to anelasticity. Units: 10^{-12}. The nominal value k_{20} for the zonal tides is taken as 0.30190. The real and imaginary parts δk_f^R and δk_f^I of δk_f are listed, along with the corresponding in-phase (*ip*) amplitude ($A_0 H_f \delta k_f^R$) and out-of-phase (*op*) amplitude ($A_0 H_f \delta k_f^I$) to be used in equation (5a).

Name	Doodson No.	deg/hr	τ	s	h	p	N'	p_s	ℓ	ℓ'	F	D	Ω	δk_f^R	Amp. (*ip*)	δk_f^I	Amp. (*op*)
	55,565	0.00221	0	0	0	0	1	0	0	0	0	0	1	0.01347	16.6	-0.00541	-6.7
	55,575	0.00441	0	0	0	0	2	0	0	0	0	0	2	0.01124	-0.1	-0.00488	0.1
S_a	56,554	0.04107	0	0	1	0	0	-1	0	-1	0	0	0	0.00547	-1.2	-0.00349	0.8
S_{sa}	57,555	0.08214	0	0	2	0	0	0	0	0	-2	2	-2	0.00403	-5.5	-0.00315	4.3
	57,565	0.08434	0	0	2	0	1	0	0	0	-2	2	-1	0.00398	0.1	-0.00313	-0.1
	58,554	0.12320	0	0	3	0	0	-1	0	-1	-2	2	-2	0.00326	-0.3	-0.00296	0.2
M_{sm}	63,655	0.47152	0	1	-2	1	0	0	1	0	0	-2	0	0.00101	-0.3	-0.00242	0.7
	65,445	0.54217	0	1	0	-1	-1	0	-1	0	0	0	-1	0.00080	0.1	-0.00237	-0.2
M_m	65,455	0.54438	0	1	0	-1	0	0	-1	0	0	0	0	0.00080	-1.2	-0.00237	3.7
	65,465	0.54658	0	1	0	-1	1	0	-1	0	0	0	1	0.00079	0.1	-0.00237	-0.2
	65,655	0.55366	0	1	0	1	0	0	1	0	-2	0	-2	0.00077	0.1	-0.00236	-0.2
M_{sf}	73,555	1.01590	0	2	-2	0	0	0	0	0	0	-2	0	-0.00009	0.0	-0.00216	0.6
	75,355	1.08875	0	2	0	-2	0	0	-2	0	0	0	0	-0.00018	0.0	-0.00213	0.3
M_f	75,555	1.09804	0	2	0	0	0	0	0	0	-2	0	-2	-0.00019	0.6	-0.00213	6.3
	75,565	1.10024	0	2	0	0	1	0	0	0	-2	0	-1	-0.00019	0.2	-0.00213	2.6
	75,575	1.10245	0	2	0	0	2	0	0	0	-2	0	0	-0.00019	0.0	-0.00213	0.2
M_{stm}	83,655	1.56956	0	3	-2	1	0	0	1	0	-2	-2	-2	-0.00065	0.1	-0.00202	0.2
M_{tm}	85,455	1.64241	0	3	0	-1	0	0	-1	0	-2	0	-2	-0.00071	0.4	-0.00201	1.1
	85,465	1.64462	0	3	0	-1	1	0	-1	0	-2	0	-1	-0.00071	0.2	-0.00201	0.5
M_{sqm}	93,555	2.11394	0	4	-2	0	0	0	0	0	-2	-2	-2	-0.00102	0.1	-0.00193	0.2
M_{qm}	95,355	2.18679	0	4	0	-2	0	0	-2	0	-2	0	-2	-0.00106	0.1	-0.00192	0.1

Table 6.3c Amplitudes ($A_2 \delta k_f H_f$) of the corrections for frequency dependence of $k_{22}^{(0)}$, taking the nominal value k_{22} for the sectorial tides as $(0.30102 - i\,0.00130)$. Units: 10^{-12}. The corrections are only to the real part.

Name	Doodson No.	deg/hr	τ	s	h	p	N'	p_s	ℓ	ℓ'	F	D	Ω	δk_f^R	Amp.
N_2	245,655	28.43973	2	-1	0	1	0	0	1	0	2	0	2	0.00006	-0.3
M_2	255,555	28.98410	2	0	0	0	0	0	0	0	2	0	2	0.00004	-1.2

6.2 Solid Earth Pole Tide

The pole tide is generated by the centrifugal effect of polar motion, characterized by the potential

$$\begin{aligned} \Delta V(r,\theta,\lambda) &= -\frac{\Omega^2 r^2}{2} \sin 2\theta \,(m_1 \cos\lambda + m_2 \sin\lambda) \\ &= -\frac{\Omega^2 r^2}{2} \sin 2\theta \; \mathbf{Re}\,[(m_1 - i m_2)\, e^{i\lambda}]. \end{aligned} \tag{10}$$

(See sub-section 7.1.4 for further details, including the relation of the wobble variables (m_1, m_2) to the polar motion variables (x_p, y_p).) The deformation which constitutes this tide produces a perturbation

$$-\frac{\Omega^2 r^2}{2} \sin 2\theta \, \mathbf{Re} \left[k_2 \left(m_1 - i m_2\right) e^{i\lambda}\right]$$

in the external potential, which is equivalent to changes in the geopotential coefficients C_{21} and S_{21}. Using for k_2 the value $0.3077 + 0.0036\, i$ appropriate to the polar tide yields

$$\begin{aligned} \Delta\bar{C}_{21} &= -1.333 \times 10^{-9}(m_1 + 0.0115 m_2), \\ \Delta\bar{S}_{21} &= -1.333 \times 10^{-9}(m_2 - 0.0115 m_1), \end{aligned}$$

where m_1 and m_2 are in seconds of arc.

6.3 Ocean Pole Tide

The ocean pole tide is generated by the centrifugal effect of polar motion on the oceans. This centrifugal effect is defined in equation (10) from section 6.2. Polar motion is dominated by the 14-month Chandler wobble and annual variations. At these long periods, the ocean pole tide is expected to have an equilibrium response, where the displaced ocean surface is in equilibrium with the forcing equipotential surface.

Desai (2002) presents a self-consistent equilibrium model of the ocean pole tide. This model accounts for continental boundaries, mass conservation over the oceans, self-gravitation, and loading of the ocean floor. Using this model, the ocean pole tide produces the following perturbations to the normalized geopotential coefficients, as a function of the wobble variables (m_1, m_2).

$$\begin{bmatrix} \Delta\bar{C}_{nm} \\ \Delta\bar{S}_{nm} \end{bmatrix} = R_n \left\{ \begin{bmatrix} \bar{A}^R_{nm} \\ \bar{B}^R_{nm} \end{bmatrix} \left(m_1\gamma_2^R + m_2\gamma_2^I\right) + \begin{bmatrix} \bar{A}^I_{nm} \\ \bar{B}^I_{nm} \end{bmatrix} \left(m_2\gamma_2^R - m_1\gamma_2^I\right) \right\} \tag{11a}$$

where

$$R_n = \frac{\Omega^2 a_E^4}{GM} \frac{4\pi G \rho_w}{g_e} \left(\frac{1 + k'_n}{2n+1}\right) \tag{11b}$$

and
Ω, a_E, GM, g_e, and G are defined in Chapter 1,
ρ_w = density of sea water = 1025 kgm^{-3},
k'_n = load deformation coefficients ($k'_2 = -0.3075, k'_3 = -0.195, k'_4 = -0.132, k'_5 = -0.1032, k'_6 = -0.0892$),
$\gamma = \gamma_2^R + i\gamma_2^I = (1 + k_2 - h_2) = 0.6870 + i0.0036$ (Values of k_2 and h_2 appropriate for the pole tide are as given in sections 6.2 and 7.1.4),
(m_1, m_2) are the wobble parameters in radians. Refer to sub-section 7.1.4 for the relationship between the wobble variables (m_1, m_2) and the polar motion variable (x_p, y_p).
The coefficients from the self-consistent equilibrium model, $\bar{A}_{nm} = \bar{A}^R_{nm} + i\bar{A}^I_{nm}$ and $\bar{B}_{nm} = \bar{B}^R_{nm} + i\bar{B}^I_{nm}$, are provided to degree and order 360 at <[2]>.

The $(n, m) = (2, 1)$ coefficients are the dominant terms of the ocean pole tide. Using the values defined above yields the following $(n, m) = (2, 1)$ coefficients for the ocean pole tide:

$$\begin{aligned} \Delta\bar{C}_{21} &= -2.2344 \times 10^{-10}(m_1 - 0.01737 m_2), \\ \Delta\bar{S}_{21} &= -1.7680 \times 10^{-10}(m_2 - 0.03351 m_1), \end{aligned}$$

where m_1 and m_2 are in seconds of arc. Approximately 90% of the variance of the ocean pole tide potential is provided by the degree $n = 2$ spherical harmonic components, with the next largest contributions provided by the degree $n = 1$ and $n = 3$ components, respectively (see Figure 6.1). Expansion to spherical harmonic degree $n = 10$ provides approximately 99% of the variance. However, adequate representation of the continental boundaries will require a spherical harmonic expansion to high degree and order. The degree $n = 1$ components are shown in Figure 6.1 to illustrate the size of the ocean pole tide contribution to geocenter motion but these terms should not be used in modeling station displacements.

[2] ftp://tai.bipm.org/iers/convupdt/chapter6/desaiscopolecoef.txt

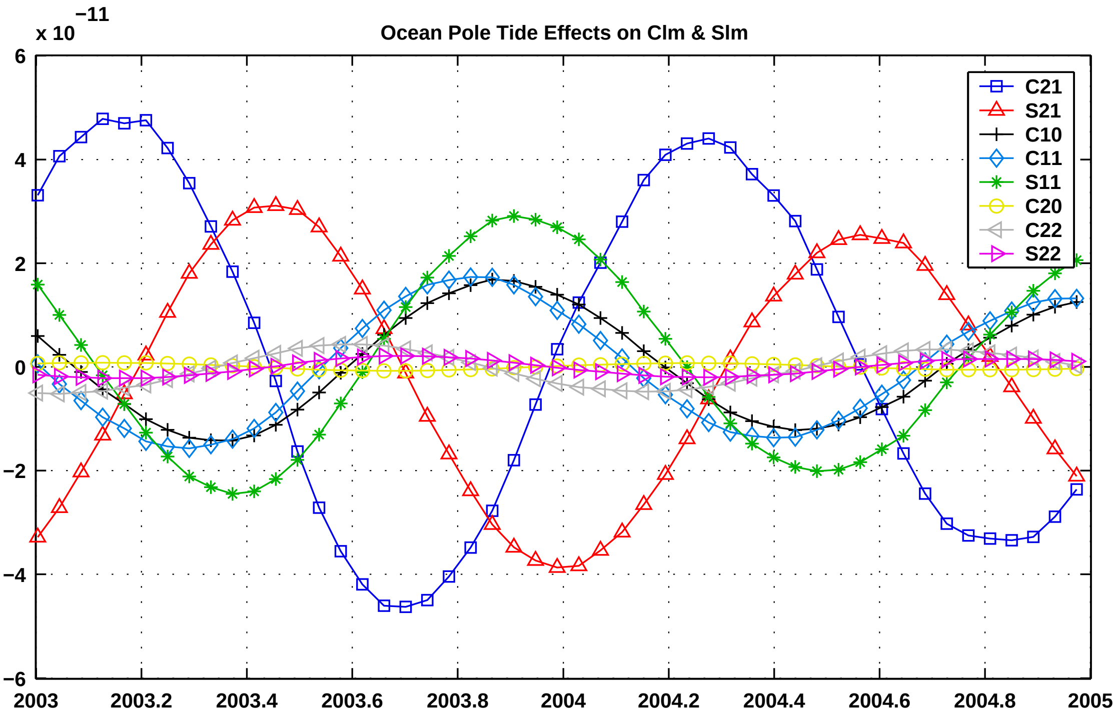

Fig. 6.1 Ocean pole tide: first spherical harmonic components.

6.4 Treatment of the Permanent Tide

The degree 2 zonal tide generating potential has a mean (time average) value that is nonzero. This time independent $(nm) = (20)$ potential produces a permanent deformation and a consequent time independent contribution to the geopotential coefficient $\bar{C}_{20}$. In formulating a geopotential model, two approaches may be taken (see Chapter 1). When the time independent contribution is included in the adopted value of $\bar{C}_{20}$, then the value is termed "zero tide" and will be noted here $\bar{C}_{20}^{zt}$. This is the case for the JGM-3 model. If the time independent contribution is not included in the adopted value of $\bar{C}_{20}$, then the value is termed "conventional tide free" and will be noted here $\bar{C}_{20}^{tf}$. This is the case of the EGM96 model.

When using the EGM96 geopotential model as originally disseminated, *i.e.* as a "conventional tide free" model, the full tidal model given by (1), computed according to the preceding sections, should be used.

In the case of a "zero tide" geopotential model, the model of tidal effects to be added should not once again contain a time independent part. One must not then use the expression (1) as it stands for modeling $\Delta\bar{C}_{20}$; its permanent part must first be restored. This is Step 3 of the computation, which provides us with $\Delta\bar{C}_{20}$.

The symbol $\Delta\bar{C}_{20}$ will hereafter be reserved for the temporally varying part of (1) while the full expression will be redesignated as $\Delta\bar{C}_{20}{}^*$ and the time independent part $\Delta\bar{C}_{20}^{perm}$. Thus

$$\Delta\bar{C}_{20} = \Delta\bar{C}_{20}^* - \langle\Delta\bar{C}_{20}{}^*\rangle, \tag{12}$$

where

$$\Delta\bar{C}_{20}^* = \frac{k_{20}}{5}\sum_{j=2}^{3}\frac{GM_j}{GM_\oplus}\left(\frac{R_e}{r_j}\right)^3 \bar{P}_{20}(\sin\Phi_j),$$

$$\langle\Delta\bar{C}_{20}^*\rangle = A_0 H_0 k_{20} = (4.4228\times10^{-8})(-0.31460)k_{20}. \tag{13}$$

When using the tidal model of section 6.1 with $k_{20} = 0.30190$, $\langle\Delta\bar{C}_{20}\rangle = -4.201\times10^{-9}$, therefore $\bar{C}_{20}^{zt} = -0.484169410\times10^{-3}$ at epoch 2000.

The use of "zero tide" values and the subsequent removal of the effect of the permanent tide from the tide model is presented for consistency with the 18th IAG General Assembly Resolution 16.

6.5 Effect of the Ocean Tides

The dynamical effects of ocean tides are most easily incorporated by periodic variations in the normalized Stokes' coefficients. These variations can be written as

$$\Delta\bar{C}_{nm} - i\Delta\bar{S}_{nm} = F_{nm} \sum_{s(n,m)} \sum_{+}^{-} (C^{\pm}_{snm} \mp iS^{\pm}_{snm}) e^{\pm i\theta_s}, \tag{14}$$

where

$$F_{nm} = \frac{4\pi G\rho_w}{g_e} \sqrt{\frac{(n+m)!}{(n-m)!(2n+1)(2-\delta_{om})}} \left(\frac{1+k'_n}{2n+1}\right),$$

g_e and G are given in Chapter 1,
ρ_w = density of seawater = 1025 kg m^{-3},
k'_n = load deformation coefficients (k'_2=−0.3075, k'_3=−0.195, k'_4=−0.132, k'_5=−0.1032, k'_6=−0.0892),
$C^{\pm}_{snm}, S^{\pm}_{snm}$ = ocean tide coefficients (m) for the tide constituent s
θ_s = argument of the tide constituent s as defined in the solid tide model (Equation 5).

Note that the index s is used in this section to identify the tide constituents while the index f is used in other parts of the document.

The summation over + and − denotes the respective addition of the retrograde waves using the top sign and the prograde waves using the bottom sign. The $C^{\pm}_{snm}$ and $S^{\pm}_{snm}$ are the coefficients of a spherical harmonic decomposition of the ocean tide height for the ocean tide due to the constituent s of the tide generating potential.

For each constituent s in the diurnal and semidiurnal tidal bands, these coefficients were obtained from the CSR 3.0 ocean tide height model (Eanes and Bettadpur, 1995), which was estimated from the TOPEX/ Poseidon satellite altimeter data. For each constituent s in the long period band, the self-consistent equilibrium tide model of Ray and Cartwright (1994) was used. The list of constituents for which the coefficients were determined was obtained from the Cartwright and Tayler (1971) expansion of the tide generating potential.

These ocean tide height harmonics are related to the Schwiderski convention (Schwiderski, 1983) according to

$$C^{\pm}_{snm} - iS^{\pm}_{snm} = -i\hat{C}^{\pm}_{snm} e^{i(\epsilon^{\pm}_{snm} + \chi_s)}, \tag{15}$$

where

$\hat{C}^{\pm}_{snm}$ = ocean tide amplitude for constituent s using the Schwiderski notation,
$\epsilon^{\pm}_{snm}$ = ocean tide phase for constituent s, and
χ_s is obtained from Table 6.4, with H_s being the Cartwright and Tayler (1971) amplitude at frequency s.

Table 6.4 Values of χ_s for long-period, diurnal and semidiurnal tides.

Tidal Band	$H_s > 0$	$H_s < 0$
Long Period	π	0
Diurnal	$\frac{\pi}{2}$	$-\frac{\pi}{2}$
Semidiurnal	0	π

For clarity, the terms in equation 14 are repeated in both conventions:

$$\Delta\bar{C}_{nm} = F_{nm} \sum_{s(n,m)} [(C^{+}_{snm} + C^{-}_{snm}) \cos\theta_s + (S^{+}_{snm} + S^{-}_{snm}) \sin\theta_s] \tag{16a}$$

or

$$\Delta\bar{C}_{nm} = F_{nm} \sum_{s(n,m)} [\hat{C}^{+}_{snm} \sin(\theta_s + \epsilon^{+}_{snm} + \chi_s) + \hat{C}^{-}_{snm} \sin(\theta_s + \epsilon^{-}_{snm} + \chi_s)], \quad (16b)$$

$$\Delta\bar{S}_{nm} = F_{nm} \sum_{s(n,m)} [(S^{+}_{snm} - S^{-}_{snm}) \cos\theta_s - (C^{+}_{snm} - C^{-}_{snm}) \sin\theta_s] \quad (16c)$$

or

$$\Delta\bar{S}_{nm} = F_{nm} \sum_{s(n,m)} [\hat{C}^{+}_{snm} \cos(\theta_s + \epsilon^{+}_{snm} + \chi_s) - \hat{C}^{-}_{snm} \cos(\theta_s + \epsilon^{-}_{snm} + \chi_s)]. \quad (16d)$$

The orbit element perturbations due to ocean tides can be loosely grouped into two classes. The resonant perturbations arise from coefficients for which the order (m) is equal to the first Doodson's argument multiplier n_1 of the tidal constituent s (See Note), and have periodicities from a few days to a few years. The non-resonant perturbations arise when the order m is not equal to index n_1. The most important of these are due to ocean tide coefficients for which $m = n_1 + 1$ and have periods of about 1 day.

Certain selected constituents (*e.g.* S_a and S_2) are strongly affected by atmospheric mass distribution (Chapman and Lindzen, 1970). The resonant harmonics (for $m = n_1$) for some of these constituents were determined by their combined effects on the orbits of several satellites. These multi-satellite values then replaced the corresponding values from the CSR 3.0 altimetric ocean tide height model.

Based on the predictions of the linear perturbation theory outlined in Casotto (1989), the relevant tidal constituents and spherical harmonics were selected for several geodetic and altimetric satellites. For geodetic satellites, both resonant and non-resonant perturbations were analyzed,whereas for altimetric satellites, only the non-resonant perturbations were analyzed. For the latter, the adjustment of empirical parameters during orbit determination removes the errors in modeling resonant accelerations. The resulting selection of ocean tidal harmonics was then merged into a single recommended ocean tide force model. With this selection the error of omission on TOPEX is approximately 5 mm along-track, and for Lageos it is 2 mm along-track. The recommended ocean tide harmonic selection is available via anonymous ftp from <[3]>.

For high altitude geodetic satellites like Lageos, in order to reduce the required computing time, it is recommended that out of the complete selection, only the constituents whose Cartwright and Tayler amplitudes H_s is greater than 0.5 mm be used, with their spherical harmonic expansion terminated at maximum degree and order 8. The omission errors from this reduced selection on Lageos is estimated at approximately 1 cm in the transverse direction for short arcs.

NOTE: The Doodson variable multipliers ($\bar{n}$) are coded into the argument number (A) after Doodson (1921) as:

$$A = n_1(n_2 + 5)(n_3 + 5).(n_4 + 5)(n_5 + 5)(n_6 + 5).$$

6.6 Conversion of Tidal Amplitudes defined according to Different Conventions

The definition used for the amplitudes of tidal terms in the recent high-accuracy tables differ from each other and from Cartwright and Tayler (1971). Hartmann and Wenzel (1995) tabulate amplitudes in units of the potential (m^2s^{-2}), while the amplitudes of Roosbeek (1996), which follow the Doodson (1921) convention, are dimensionless. To convert them to the equivalent tide heights H_f of the Cartwright-Tayler convention, multiply by the appropriate factors from Table 6.5. The following values are used for the constants appearing in the conversion factors: Doodson constant $D_1 = 2.63358352855$ m^2 s^{-2}; $g_e \equiv g$ at the equatorial radius = 9.79828685 (from $GM = 3.986004415 \times 10^{14}$ m^3 s^{-2}, $R_e = 6378136.55$ m).

[3]ftp.csr.utexas.edu/pub/tide

Table 6.5 Factors for conversion to Cartwright-Tayler amplitudes from those defined according to Doodson's and Hartmann and Wenzel's conventions.

From Doodson	From Hartmann & Wenzel
$f_{20} = -\frac{\sqrt{4\pi}}{\sqrt{5}}\frac{D_1}{g_e} = -0.426105$	$f'_{20} = \frac{2\sqrt{\pi}}{g_e} = 0.361788$
$f_{21} = -\frac{2\sqrt{24\pi}}{3\sqrt{5}}\frac{D_1}{g_e} = -0.695827$	$f'_{21} = -\frac{\sqrt{8\pi}}{g_e} = -0.511646$
$f_{22} = \frac{\sqrt{96\pi}}{3\sqrt{5}}\frac{D_1}{g_e} = 0.695827$	$f'_{22} = \frac{\sqrt{8\pi}}{g_e} = 0.511646$
$f_{30} = -\frac{\sqrt{20\pi}}{\sqrt{7}}\frac{D_1}{g_e} = -0.805263$	$f'_{30} = \frac{2\sqrt{\pi}}{g_e} = 0.361788$
$f_{31} = \frac{\sqrt{720\pi}}{8\sqrt{7}}\frac{D_1}{g_e} = 0.603947$	$f'_{31} = \frac{\sqrt{8\pi}}{g_e} = 0.511646$
$f_{32} = \frac{\sqrt{1440\pi}}{10\sqrt{7}}\frac{D_1}{g_e} = 0.683288$	$f'_{32} = \frac{\sqrt{8\pi}}{g_e} = 0.511646$
$f_{33} = -\frac{\sqrt{2880\pi}}{15\sqrt{7}}\frac{D_1}{g_e} = -0.644210$	$f'_{33} = -\frac{\sqrt{8\pi}}{g_e} = -0.511646$

References

Cartwright, D. E. and Tayler, R. J., 1971, "New Computations of the Tide-Generating Potential," *Geophys. J. Roy. astr. Soc.*, **23**, pp. 45–74.

Casotto, S., 1989, "Ocean Tide Models for TOPEX Precise Orbit Determination," Ph.D. Dissertation, The Univ. of Texas at Austin.

Chapman, S. and Lindzen, R., 1970, *Atmospheric Tides*, D. Reidel, Dordrecht, 200 pp.

Chao, B. F., Ray, R. D., Gipson, J. M., Egbert, G. D. and Ma, C., 1996, "Diurnal/semidiurnal polar motion excited by oceanic tidal angular momentum," *J. Geophys. Res.*, **101**, pp. 20151–20163.

Desai, S. and Wahr, J. M., 1995, "Empirical ocean tide models estimated from Topex/Poseidon altimetry," *J. Geophys. Res.*, **100**, pp. 25205–25228.

Desai, S. D., 2002, "Observing the pole tide with satellite altimetry," *J. Geophys. Res.*, **107**(C11), 3186, doi:10.1029/ 2001JC001224.

Doodson, A. T., 1921, "The Harmonic Development of the Tide-Generating Potential," *Proc. R. Soc. A.*, **100**, pp. 305–329.

Eanes, R. J., Schutz, B., and Tapley, B., 1983, "Earth and Ocean Tide Effects on Lageos and Starlette," in *Proc. of the Ninth International Symposium on Earth Tides*, Kuo, J. T. (ed.), E. Sckweizerbart'sche Verlagabuchhandlung, Stuttgart.

Eanes R. J. and Bettadpur, S., 1995, "The CSR 3.0 global ocean tide model," *Technical Memorandum CSR-TM-95-06*, Center for Space Research, University of Texas, Austin, TX.

Gruber, Th., Bode, A., Reigber, Ch., Schwintzer, P., Balmino, G., Biancale, R., and Lemoine, J.-M., 2000, *Geophys. Res. Lett.*, **27**, pp. 4005–4008.

Hartmann, T. and Wenzel, H.-G., 1995, "The HW95 Tidal Potential Catalogue," *Geophys. Res. Lett.*, **22**, pp. 3553–3556.

Lemoine, F. G., Kenyon, S. C., Factor, J. K., Trimmer, R. G., Pavlis, N. K., Chinn, D. S., Cox, C. M., Klosko, S. M., Luthke, S. B., Torrance, M. H., Wang, Y. M., Williamson, R. G., Pavlis, E. C., Rapp, R. H., and Olson, T. R., 1998, "The Development of the Joint NASA GSFC and National Imagery and Mapping Agency (NIMA) Geopotential Model EGM96," *NASA/TP-1998-206861*, Goddard Space Flight Center, Greenbelt, Maryland.

Mathews, P. M., Buffett, B. A., and Shapiro,. I. I., 1995, "Love numbers for diurnal tides: Relation to wobble admittances and resonance expansions," *J. Geophys. Res.*, **100**, pp. 9935–9948.

Mathews, P. M., Herring, T. A., and Buffett, B. A., 2002, "Modeling of nutation-precession: New nutation series for nonrigid Earth, and insights into the Earth's interior," *J. Geophys. Res.*, **107, B4**, 10.1029/2001JB000390.

Ray, R. D. and Cartwright, D. E., 1994, "Satellite altimeter observations of the M_f and M_m ocean tides, with simultaneous orbit corrections," *Gravimetry and Space Techniques Applied to Geodynamics and Ocean Dynamics*, Geophysical Monograph 82, IUGG Volume 17, pp. 69–78.

Roosbeek, F., 1996, "RATGP95: a harmonic development of the tide-generating potential using an analytical method," *Geophys. J. Int.*, **126**, pp. 197–204.

Schwiderski, E., 1983, "Atlas of Ocean Tidal Charts and Maps, Part I: The Semidiurnal Principal Lunar Tide M2," *Marine Geodesy*, **6**, pp. 219–256.

Souchay, J. and Folgueira, M., 2000, "The effect of zonal tides on the dynamical ellipticity of the Earth and its influence on the mutation," *Earth, Moon and Planets*, **81**, pp. 201–216.

Tapley, B. D., Watkins, M. M., Ries, J. C., Davis, G. W., Eanes, R. J., Poole, S. R., Rim, H. J., Schutz, B. E., Shum, C. K., Nerem, R. S., Lerch, F. J., Marshall, J. A., Klosko, S. M., Pavlis, N. K., and Williamson, R. G., 1996, "The Joint Gravity Model 3," *J. Geophys. Res*, **101**, pp. 28029–28049.

Wahr, J. M., 1981, "The Forced Nutations of an Elliptical, Rotating, Elastic, and Oceanless Earth," *Geophys. J. Roy. Astron. Soc.*, **64**, pp. 705–727.

Wahr, J., 1987, "The Earth's C_{21} and S_{21} gravity coefficients and the rotation of the core," *Geophys. J. Roy. astr. Soc.*, **88**, pp. 265–276.

Wahr, J. M. and Sasao, T., 1981, "A diurnal resonance in the ocean tide and the Earth's load response due to the resonant free "core nutation"," *Geophys. J. R. astr. Soc.*, **64**, pp. 747–765.

Wahr, J., 1990, "Corrections and Update to 'The Earth's C_{21} and S_{21} gravity coefficients and the rotation of the core'," *Geophys. J. Int.*, **101**, pp. 709–711.

Wahr, J. and Bergen, Z., 1986, "The effects of mantle elasticity on nutations, Earth tides, and tidal variations in the rotation rate," *Geophys. J. R. astr. Soc.*, **87**, 633–668.

Widmer, R., Masters, G., and Gilbert, F., 1991, "Spherically symmetric attenuation within the Earth from normal mode data," *Geophys. J. Int.*, **104**, pp. 541–553.