9 Tropospheric Model (22 June 2007)

9.1 Optical Techniques

The accuracy of satellite and lunar laser ranging (SLR & LLR) is greatly affected by the residual errors in modeling the effect of signal propagation through the troposphere and stratosphere. Although several models for atmospheric correction have been developed, the more traditional approach in LR data analysis uses a model developed in the 1970s (Marini and Murray, 1973). Mendes *et al.* (2002) pointed out some limitations in that model, namely the modeling of the elevation dependence of the zenith atmospheric delay, *i.e.* the mapping function (MF) component of the model. The MFs developed by Mendes *et al.* (2002) represent a significant improvement over the MF in the Marini-Murray model and other known MFs. Of particular interest is the ability of the new MFs to be used in combination with any zenith delay (ZD) model to predict the atmospheric delay in the line-of-sight direction. Subsequently, Mendes and Pavlis (2004) developed a more accurate ZD model, applicable to the range of wavelengths used in modern LR instrumentation. The combined set of the new mapping function and the new ZD model were adopted in October 2006 by the Analysis Working Group of the International Laser Ranging Service (ILRS) as the new standard model to be used for the analysis of LR data starting January 1, 2007. The alternative to correct the atmospheric delay using two-color ranging systems is still at an experimental stage.

9.1.1 Zenith Delay Models

The atmospheric propagation delay experienced by a laser signal in the zenith direction is defined as

$$d^z_{atm} = 10^{-6} \int_{r_s}^{r_a} N dz = \int_{r_s}^{r_a} (n-1)\, dz, \tag{1}$$

or, if we split the zenith delay into hydrostatic (d^z_h) and non-hydrostatic (d^z_{nh}) components,

$$d^z_{atm} = d^z_h + d^z_{nh} = 10^{-6} \int_{r_s}^{r_a} N_h dz + 10^{-6} \int_{r_s}^{r_a} N_{nh} dz, \tag{2}$$

where $N = (n-1) \times 10^6$ is the (total) group refractivity of moist air, n is the (total) refractive index of moist air, N_h and N_{nh} are the hydrostatic and the non-hydrostatic components of the refractivity, r_s is the geocentric radius of the laser station, r_a is the geocentric radius of the top of the (neutral) atmosphere, and d^z_{atm} and dz have length units.

In the last few years, the computation of the group refractivity at optical wavelengths has received special attention and, as a consequence, the International Association of Geodesy (IAG) (IUGG, 1999) recommended a new procedure to compute the group refractivity, following Ciddor (1996) and Ciddor and Hill (1999). Based on this procedure, Mendes and Pavlis (2004) derived closed-form expressions to compute the zenith delay. For the hydrostatic component, we have

$$d^z_h = 0.002416579 \frac{f_h(\lambda)}{f_s(\phi, H)} P_s, \tag{3}$$

where d^z_h is the zenith hydrostatic delay, in meters, and P_s is the surface barometric pressure, in hPa. The function $f_s(\phi, H)$ is given by

$$f_s(\phi, H) = 1 - 0.00266 \cos 2\phi - 0.00000028 H, \tag{4}$$

where ϕ is the geodetic latitude of the station and H is the geodetic height of the station in meters <[1]>, $f_h(\lambda)$ is the dispersion equation for the hydrostatic component

[1] originally, Saastamoinen (1972) used orthometric height, however, the formula is insensitive to the difference, so geodetic height can be used instead without loss of accuracy.

$$f_h(\lambda) = 10^{-2} \times \left[k_1^* \frac{(k_0 + \sigma^2)}{(k_0 - \sigma^2)^2} + k_3^* \frac{(k_2 + \sigma^2)}{(k_2 - \sigma^2)^2} \right] C_{CO_2}, \tag{5}$$

with $k_0 = 238.0185$ μm^{-2}, $k_2 = 57.362$ μm^{-2}, $k_1^* = 19990.975$ μm^{-2}, and $k_3^* = 579.55174$ μm^{-2}, σ is the wave number ($\sigma = \lambda^{-1}$, where λ is the wavelength, in μm), $C_{CO_2} = 1 + 0.534 \times 10^{-6}(x_c - 450)$, and x_c is the carbon dioxide (CO_2) content, in ppm. In the conventional formula, a CO_2 content of 375 ppm should be used, in line with the IAG recommendations, thus $C_{CO_2} = 0.99995995$ should be used.

For the non-hydrostatic component, we have:

$$d_{nh}^z = 10^{-4}(5.316 f_{nh}(\lambda) - 3.759 f_h(\lambda)) \frac{\mathrm{e_s}}{f_s(\phi, H)}, \tag{6}$$

where d_{nh}^z is the zenith non-hydrostatic delay, in meters, and e_s is the surface water vapor pressure, in hPa. f_{nh} is the dispersion formula for the non-hydrostatic component:

$$f_{nh}(\lambda) = 0.003101\left(\omega_0 + 3\omega_1\sigma^2 + 5\omega_2\sigma^4 + 7\omega_3\sigma^6\right), \tag{7}$$

where $\omega_0 = 295.235$, $\omega_1 = 2.6422$ μm^2, $\omega_2 = -0.032380$ μm^4, and $\omega_3 = 0.004028$ μm^6.

The subroutine `fcul_ztd_hPa.f` to compute the total zenith delay is available at <[2]>.

From the assessment of the zenith models against ray tracing for the most used wavelengths in LR, it can be concluded that these zenith delay models have overall rms errors for the total zenith delay below 1 mm across the whole frequency spectrum (Mendes and Pavlis, 2003; Mendes and Pavlis, 2004).

9.1.2 Mapping Function

Due to the small contribution of water vapor to atmospheric refraction at visible wavelengths, we can consider a single MF for laser ranging. In this case, we have:

$$d_{atm} = d_{atm}^z \cdot m(e), \tag{8}$$

where d_{atm}^z is the total zenith propagation delay and $m(e)$ the (total) MF. Mendes *et al.* (2002) derived a MF, named FCULa, based on a truncated form of the continued fraction in terms of $1/sin(e)$ (Marini, 1972), normalized to unity at the zenith

$$m(e) = \frac{1 + \cfrac{a_1}{1 + \cfrac{a_2}{1 + a_3}}}{\sin e + \cfrac{a_1}{\sin e + \cfrac{a_2}{\sin e + a_3}}}. \tag{9}$$

Note that the same formula is used for radio techniques, but with different variables, see equation (13). The FCULa MF is based on ray tracing through one full year of radiosonde data from 180 globally distributed stations. It is valid for a wide range of wavelengths from 0.355 μm to 1.064 μm (Mendes and Pavlis, 2003) and for elevation angles greater than 3 degrees, if we neglect the contribution of horizontal refractivity gradients. The coefficients a_i (i=1,2,3) have the following mathematical formulation:

[2] ftp://tai.bipm.org/iers/convupdt/chapter9

Table 1: Coefficients (a_{ij}) for the FCULa mapping function, see equation (10). Coefficients (a_{i1}) are in C^{-1} and coefficients (a_{i3}) in m^{-1}.

a_{ij}	FCULa
a_{10}	$(12100.8 \pm 1.9) \times 10^{-7}$
a_{11}	$(1729.5 \pm 4.3) \times 10^{-9}$
a_{12}	$(319.1 \pm 3.1) \times 10^{-7}$
a_{13}	$(-1847.8 \pm 6.5) \times 10^{-11}$
a_{20}	$(30496.5 \pm 6.6) \times 10^{-7}$
a_{21}	$(234.6 \pm 1.5) \times 10^{-8}$
a_{22}	$(-103.5 \pm 1.1) \times 10^{-6}$
a_{23}	$(-185.6 \pm 2.2) \times 10^{-10}$
a_{30}	$(6877.7 \pm 1.2) \times 10^{-5}$
a_{31}	$(197.2 \pm 2.8) \times 10^{-7}$
a_{32}	$(-345.8 \pm 2.0) \times 10^{-5}$
a_{33}	$(106.0 \pm 4.2) \times 10^{-9}$

$$a_i = a_{i0} + a_{i1} t_s + a_{i2} \cos\phi + a_{i3} H, \tag{10}$$

where t_s is the temperature at the station in Celsius degrees, H is the geodetic height of the station, in meters, and the coefficients are given in Table 1, see Mendes *et al.* (2002) for details. The subroutine `FCUL_a.f` to compute the FCULa mapping function is available at <[2]>.

The new mapping functions represent a significant improvement over other mapping functions available and have the advantage of being easily combined with different zenith delay models. The analysis of two years of SLR data from LAGEOS and LAGEOS 2 indicate a clear improvement in the estimated station heights (8% reduction in variance), while the simultaneously adjusted tropospheric zenith delay biases were all consistent with zero (Mendes *et al.*, 2002).

For users who do not have extreme accuracy requirements or do not know the station temperature, the FCULb mapping function, which depends on the station location and the day of the year, has been developed, see Mendes *et al.* (2002) for details. The subroutine `FCUL_b.f` to compute the FCULb mapping function is available at <[2]>.

9.1.3 Future Developments

The accuracy of the new atmospheric delay models are still far from the accuracy required for global climate change studies. The goal as set forth by the International Laser Ranging Service (ILRS) is better than one millimeter. The LR community has been looking into ways to achieve that accuracy. One significant component that is missing from the above models is to account for the effect of horizontal gradients in the atmosphere, an error source that contributes up to 5 cm of delay at low elevation angles. Ranging at low elevation angles improves the de-correlation of errors in the vertical coordinate with errors in the measurement process (biases). Stations thus strive to range as low as possible, thence the need for model improvements.

Global meteorological fields are now becoming more readily accessible, with higher spatio-temporal resolution, better accuracy and more uniform quality. This is primarily due to the availability of satellite observations with global coverage twice daily. Hulley and Pavlis (2007) developed a new technique, and tested it with real data, computing the total atmospheric delay, including horizontal gradients, via three-dimensional atmospheric ray tracing (3D ART) with meteorological fields from the Atmospheric Infrared Sounder (AIRS). This technique has already been tested and applied to two years of SLR data from LAGEOS 1 and 2, and for ten core, globally-distributed SLR stations. Replacing the atmospheric corrections estimated from the Mendes-Pavlis ZD and MF models with 3D ART resulted in reducing the variance of the SLR range residuals by up to 25% for all the data used in the analysis. As of May 2007, an effort is in progress to establish a service that will compute these corrections for all of the collected SLR and LLR data in the future. Once this service is in place, it is expected that this new approach will be adopted as the standard for SLR and LLR data reductions.

9.2 Radio Techniques

The non-dispersive delay imparted by the atmosphere on a radio signal up to 30 GHz in frequency, which reaches a magnitude of about 2.3 m at sea level, is conveniently divided into "hydrostatic" and "wet" components. The hydrostatic delay is caused by the refractivity of the dry gases (mainly N_2 and O_2) in the troposphere and by most of the nondipole component of the water vapor refractivity. The rest of the water vapor refractivity is responsible for most of the wet delay. The hydrostatic delay component accounts for roughly 90% of the total delay at any given site globally, but can vary between about 80 and 100% depending on location and time of year. It can be accurately computed *a priori* based on reliable surface pressure data using the formula of Saastamoinen (1972) as given by Davis *et al.* (1985):

$$D_{hz} = \frac{[(0.0022768 \pm 0.0000005)]P_0}{f_s(\phi, H)} \tag{11}$$

where D_{hz} is the zenith hydrostatic delay in meters, P_0 is the total atmospheric pressure in hPa (equivalent to millibars) at the antenna reference point (*e.g.* antenna phase center for GPS, the intersection of the axes of rotation for VLBI [3]), and the function $f_s(\phi, H)$ is given in equation (4).

There is currently no simple method to estimate an accurate *a priori* value for the wet tropospheric delay, although research continues into the use of external monitoring devices (such as water vapor radiometers) for this purpose. So, in most precise applications where sub-decimeter accuracy is sought, the residual delay must usually be estimated with the other geodetic quantities of interest. The estimation is facilitated by a simple parameterization of the tropospheric delay, where the line-of-sight delay, D_L, is expressed as a function of four parameters as follows:

$$D_L = m_h(e)D_{hz} + m_w(e)D_{wz} + m_g(e)[G_N \cos(a) + G_E \sin(a)]. \tag{12}$$

The four parameters in this expression are the zenith hydrostatic delay, D_{hz}, the zenith wet delay, D_{wz}, and a horizontal delay gradient with components G_N and G_E. m_h, m_w and m_g are the hydrostatic, wet, and gradient mapping functions, respectively, and e is the elevation angle of the observed radio source in vacuum. a is the azimuth angle in which the signal is received, measured east from north. Horizontal gradients are needed to account for a systematic component in the N/S direction towards the equator due to the atmospheric bulge and also for random components in both directions due to weather systems. Horizontal tropospheric gradients can reach or exceed 1 mm and their estimation was shown by Chen and Herring (1997) and MacMillan (1995) to be beneficial in VLBI, and by Bar-Sever *et al.* (1998) to be beneficial in GPS. Davis *et al.* (1993) and MacMillan (1995) recommend using either $m_g(e) = m_h(e)\cot(e)$ or $m_g(e) = m_w(e)\cot(e)$. Chen and Herring (1997) propose using $m_g(e) = 1/(\sin e \tan e + 0.0032)$. The various forms agree to within 10% for elevation angles higher than 10°, but the differences reach 50% for 5° elevation due to the singularity of the $\cot(e)$ function. The estimation of gradients is only worthwhile when using data lower than 15° in elevation. In the case of GPS analyses, such low-elevation data could be deweighted because of multipath effects.

The hydrostatic and wet mapping functions, m_h and m_w, for the neutral atmosphere depend on the vertical distribution of the hydrostatic and wet refractivity above the geodetic sites. With the availability of numerical weather models (NWM) this information can currently be extracted globally with a temporal resolution of six hours (Niell, 2001). Unlike previous mapping functions these are not limited in their accuracy by the use of only surface meteorological data, as in the

[3] In the case of VLBI, provision should be made to account for the actual path of the photons due to the possible altitude variation of the reference point (Sovers and Jacobs, 1996)

functions of Lanyi (1984), Ifadis (1986) or in MTT (Herring, 1992), nor by the use of average *in situ* properties of the atmosphere, even if validated with radiosonde data, as in NMF (Niell, 1996). The general form of the hydrostatic and wet mapping functions is (Herring, 1992)

$$m_{h,w}\left(e\right)=\frac{1+\dfrac{a}{1+\dfrac{b}{1+c}}}{\sin e+\dfrac{a}{\sin e+\dfrac{b}{\sin e+c}}}. \tag{13}$$

The Vienna Mapping Function 1 (VMF1) (Boehm *et al.*, 2006a) is based on exact ray traces through the refractivity profiles of a NWM at 3° elevation and empirical equations for the b and c coefficients of the continued fraction in equation (13). Niell (2006) compared mapping functions determined from radiosonde data in 1992 with VMF1 and found that the equivalent station height standard deviations are less than 3 mm, which is significantly better than for other mapping functions available. These results are confirmed by VLBI analyses as shown by Boehm *et al.* (2007a) and Tesmer *et al.* (2007), respectively. Thus, VMF1 is recommended for any global application, such as the determination of the terrestrial reference frame and Earth orientation parameters.

At the webpage <[4]>, the a coefficients of VMF1 as derived from data of the European Centre for Medium-Range Weather Forecasts (ECMWF) are provided with a time interval of 6 hours for the positions of all sites of the International GNSS Service (IGS), the International VLBI Service for Geodesy and Astrometry (IVS), and the International DORIS Service (IDS), as well as on a global $2.5° \times 2.0°$ grid. Kouba (2007) compares results from the grids with VMF1 given at the sites, and he provides algorithms on how to use the grids.

The Global Mapping Function (GMF) (Boehm *et al.*, 2006b) is an empirical mapping function in the tradition of NMF that can be calculated using only station latitude, longitude (not used by NMF), height, and day of the year. GMF, which is based on spherical harmonics up to degree and order 9, was developed with the goal to be more accurate than NMF and to be consistent with VMF1. Some comparisons of GMF, VMF1 and other MFs with radiosonde data may be found in (Niell, 2006). GMF is easy to implement and can be used when the best accuracy is not required or when VMF1 is not available. The Fortran subroutine `gmf.f` is available at <[2]> and <[4]>.

9.3 Sources for meteorological data

Because 1 mbar pressure error causes an *a priori* delay error of about 2.3 mm at sea level, it is essential to use accurate estimates of meteorological data (Tregoning and Herring, 2006). If meteorological instrumentation is not available, meteorological data may be retrieved from a NWM, *e.g.* the ECMWF as provided together with VMF1 at <[4]>. In both cases adjustments of the pressure should be applied for the height difference between the location of the pressure measurement (from *in situ* instrumentation or from NWM) and the reference point of the space geodesy instrument. Commonly used formulas for the adjustment can be found in (Boehm *et al.*, 2007b). Alternatively, local pressure and temperature estimates could be determined with the empirical model GPT (Boehm *et al.*, 2007b) that has been developed similarly to the GMF, and is provided as a Fortran routine, `gpt.f`, at <[2]> and <[4]>.

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