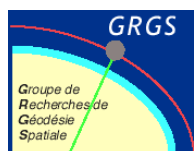


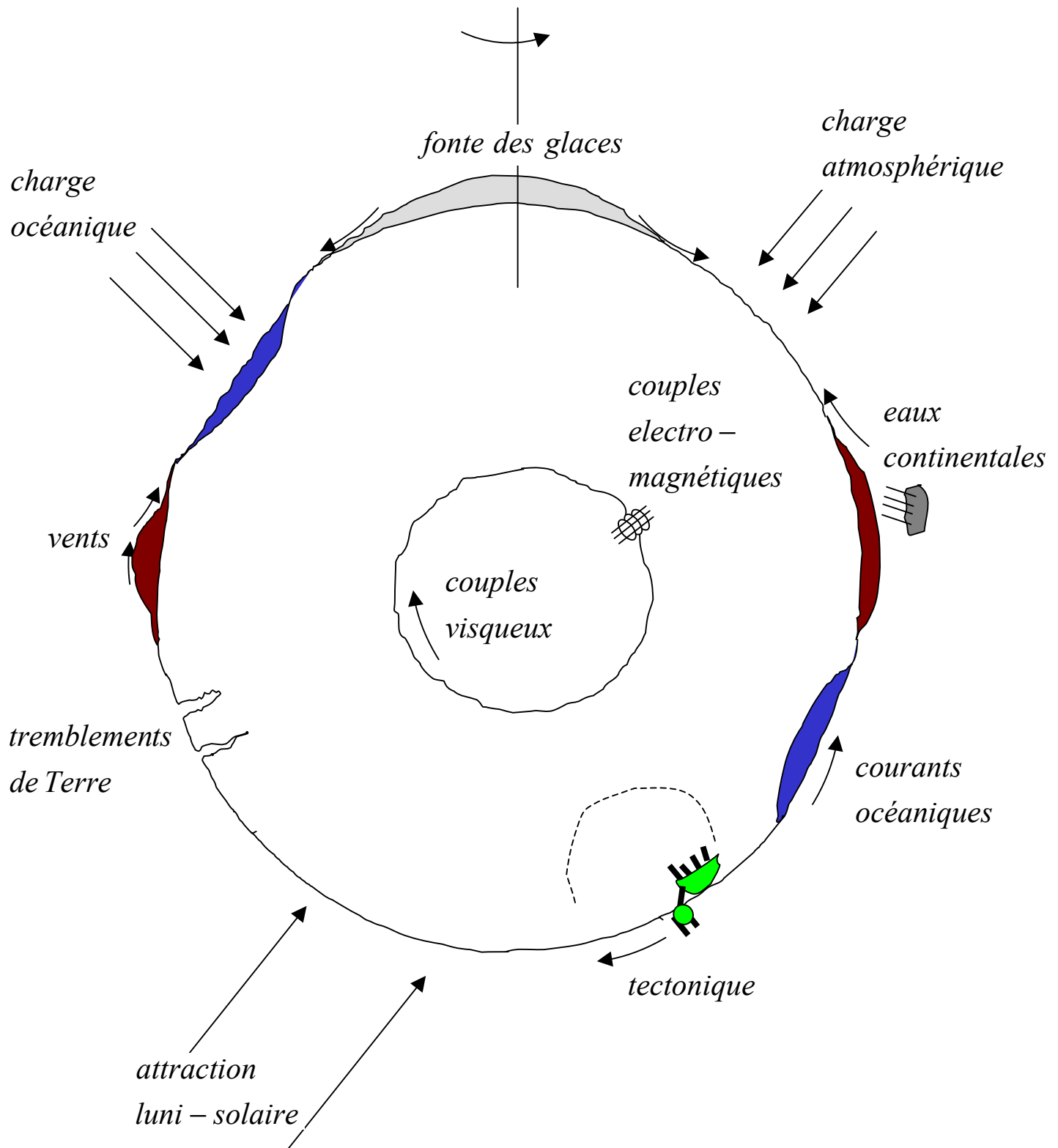
# Petits mouvements des stations

*Richard BIANCALE, CNES DSO/ED/GS/GTP*



Ecole de Géodésie Spatiale 2-6 Septembre 2002 Forcalquier

# La Terre se déforme



# Les masses

## Masse de la Terre :

$$G = 6,672 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

$$GM = 3,986004415 \cdot 10^{14} \text{ m}^3 \text{ s}^{-2}$$

$$M = 5,97 \cdot 10^{24} \text{ kg}$$

## Masse de l'océan :

$$M_o = \rho_w (4 \pi R^2) h_m \cdot 71\%$$

$$= 1025 \cdot 5,10 \cdot 10^{14} \cdot 3800 \cdot 0,71$$

$$= 1,41 \cdot 10^{21} \text{ kg}$$

*soit 0,24 ‰ de la masse de la Terre*

## Masse de l'atmosphère :

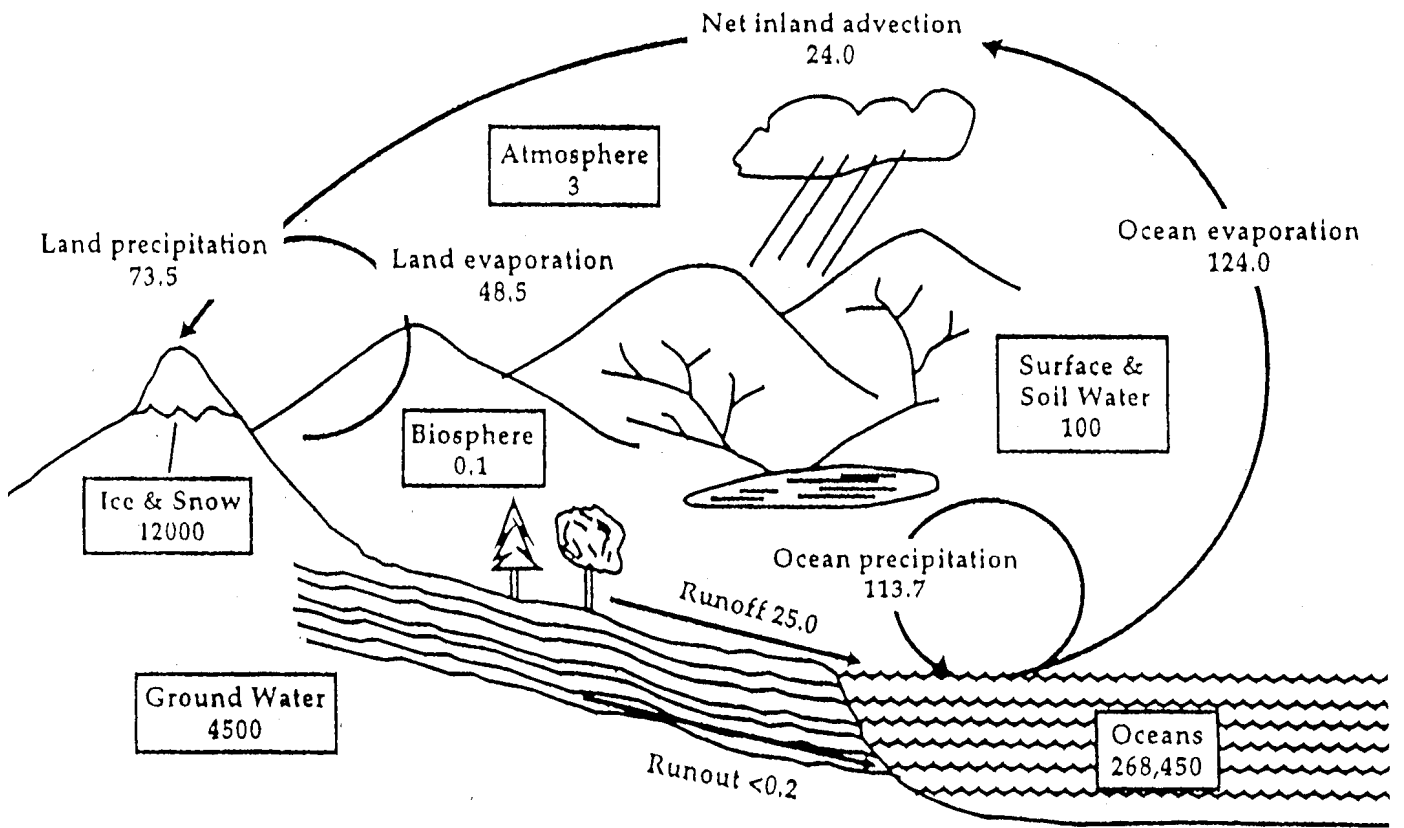
$$P_a = 101325 \text{ Pa} \quad (\text{kg m}^{-1} \text{ s}^{-2} = \text{N m}^{-2})$$

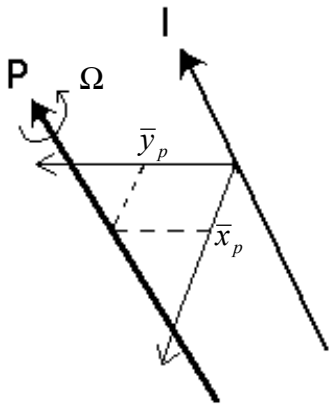
$$M_a = (4 \pi R^2) P_a / g$$

$$= 5,10 \cdot 10^{14} \cdot 101325 / 9,80$$

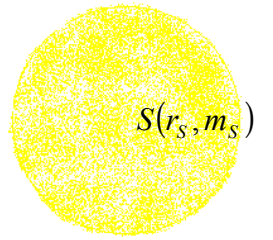
$$= 5,27 \cdot 10^{18} \text{ kg}$$

*soit 3,7 ‰ de la masse de l'océan*

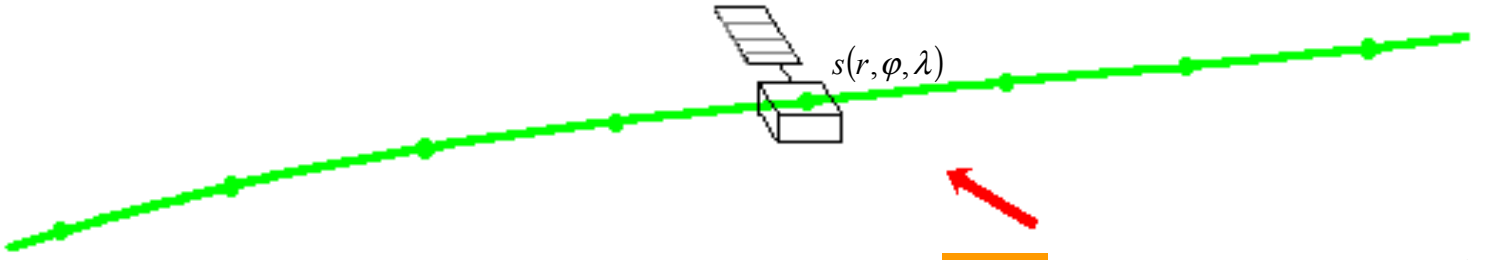




$$\begin{Bmatrix} \Delta C_{21} \\ \Delta S_{21} \end{Bmatrix} = \begin{bmatrix} k_2 \\ k_0 \end{bmatrix} C_{20} \begin{Bmatrix} x_p \\ -y_p \end{Bmatrix}$$

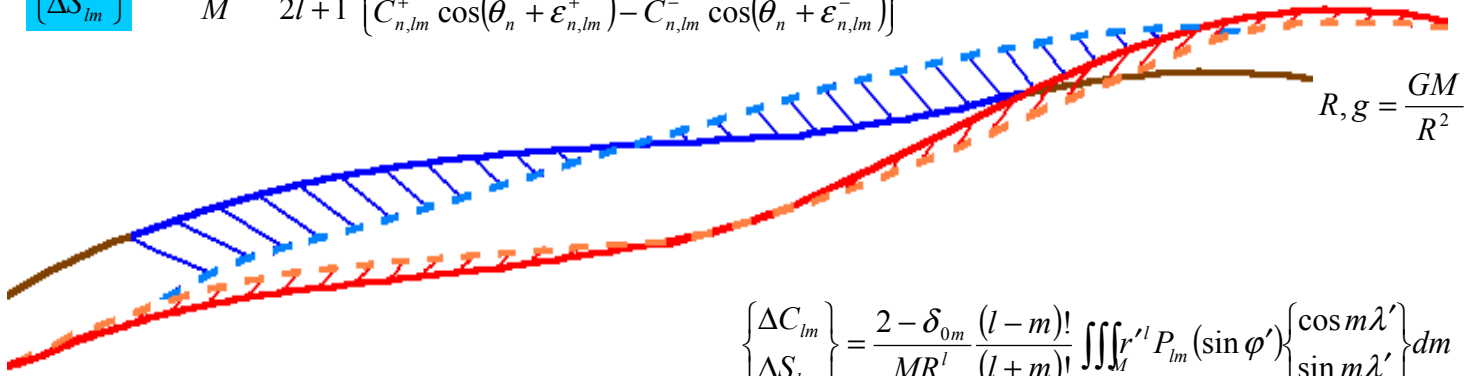


$$\begin{Bmatrix} \Delta C_{lm} \\ \Delta S_{lm} \end{Bmatrix} = (2 - \delta_{0m}) \frac{(l-m)! Gm_s}{(l+m)! GM} \left(\frac{r}{r_s}\right)^{l+1} \left(\frac{r}{R}\right)^l \times P_m(\sin \varphi_s) \begin{Bmatrix} \cos m\lambda_s \\ \sin m\lambda_s \end{Bmatrix}$$

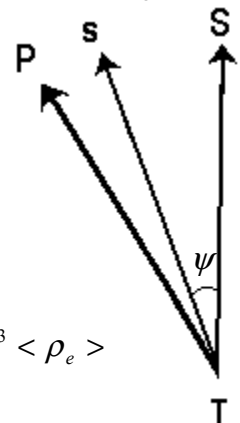


$$\begin{Bmatrix} \Delta C_{lm} \\ \Delta S_{lm} \end{Bmatrix} = k_l (2 - \delta_{0m}) \frac{(l-m)! Gm_s}{(l+m)! GM} \left(\frac{R}{r_s}\right)^{l+1} \times P_m(\sin \varphi_s) \begin{Bmatrix} \cos m\lambda_s \\ \sin m\lambda_s \end{Bmatrix}$$

$$\begin{Bmatrix} \Delta C_{lm} \\ \Delta S_{lm} \end{Bmatrix} = \frac{4\pi R^2 \rho_w}{M} \frac{1 + k_l'}{2l+1} \begin{Bmatrix} C_{n,lm}^+ \sin(\theta_n + \varepsilon_{n,lm}^+) + C_{n,lm}^- \sin(\theta_n + \varepsilon_{n,lm}^-) \\ C_{n,lm}^+ \cos(\theta_n + \varepsilon_{n,lm}^+) - C_{n,lm}^- \cos(\theta_n + \varepsilon_{n,lm}^-) \end{Bmatrix}$$



$$\begin{Bmatrix} \Delta C_{lm} \\ \Delta S_{lm} \end{Bmatrix} = \frac{2 - \delta_{0m}}{MR^l} \frac{(l-m)!}{(l+m)!} \iint r'^l P_m(\sin \varphi') \begin{Bmatrix} \cos m\lambda' \\ \sin m\lambda' \end{Bmatrix} dm$$



$$M = \frac{4}{3} \pi R^3 \langle \rho_e \rangle$$

# Déformation de marée terrestre

D'après la théorie de Love, les déformations de marée terrestre sont proportionnelles au potentiel perturbateur luni-solaire :

$$u_r = \sum_{\ell=2}^3 \frac{h_\ell}{g} \Delta U_\ell$$

$$u_\varphi = \sum_{\ell=2}^3 \frac{\ell_\ell}{g} \frac{\partial \Delta U_\ell}{\partial \varphi}$$

$$u_\lambda = \sum_{\ell=2}^3 \frac{\ell_\ell}{g \cos \varphi} \frac{\partial \Delta U_\ell}{\partial \lambda}$$

$h_\ell$  : nombre de Love de déplacement vertical

$\ell_\ell$  : nombre de Shida de déplacement horizontal

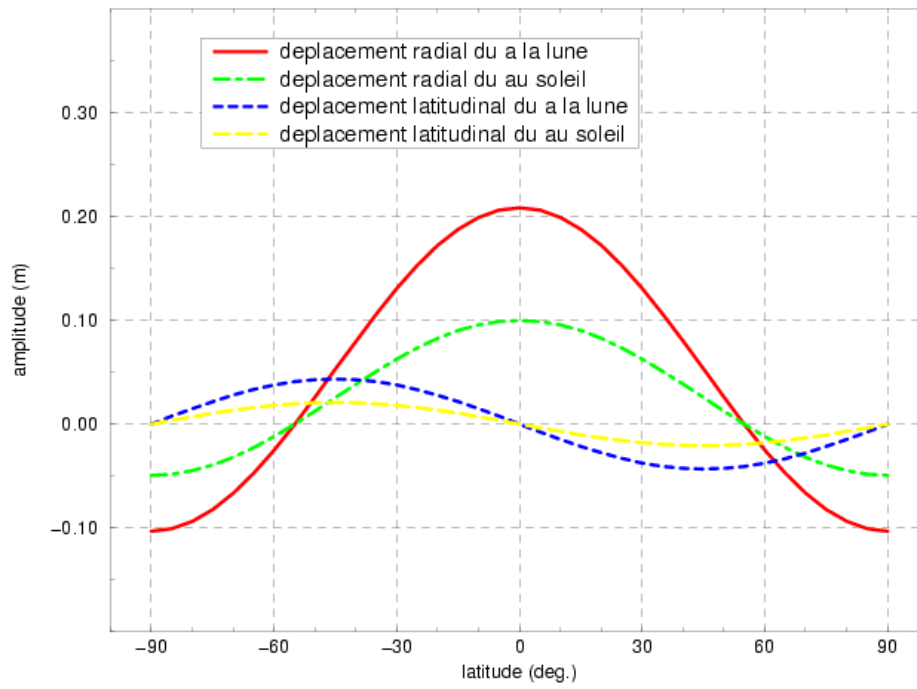
soit en coordonnées rectangulaires ; compte tenu du développement du potentiel de marée terrestre :

$$\begin{aligned} \overline{\Delta R} = & \frac{Gmp}{GM} \frac{R^4}{r_p^3} \left\{ 3 \ell_2 \cos \psi \frac{\overline{r_p}}{r_p} + \left[ 3 \left( \frac{h_2}{2} - \ell_2 \right) \cos^2 \psi - \frac{h_2}{2} \right] \frac{\overline{R}}{R} \right\} \\ & + \frac{Gmp}{GM} \frac{R^5}{r_p^4} \left\{ \ell_3 \frac{15 \cos^2 \psi - 3}{2} \frac{\overline{r_p}}{r_p} + \frac{3 \cos \psi}{2} \left[ 5 \left( \frac{h_3}{3} - \ell_3 \right) \cos^2 \psi + \ell_3 - h_3 \right] \frac{\overline{R}}{R} \right\} \end{aligned}$$

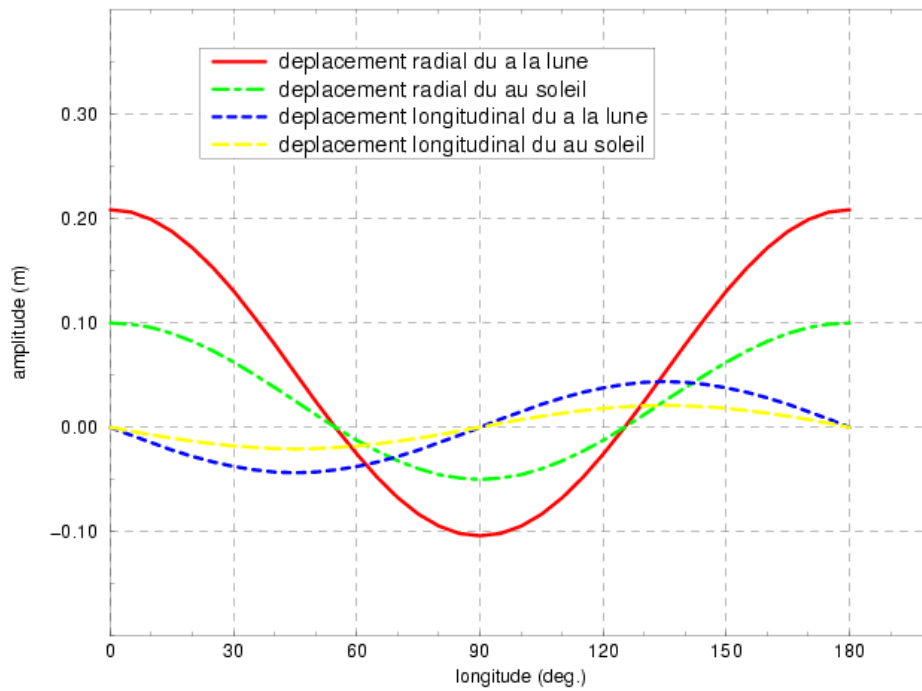
avec  $\cos \psi = \frac{\overline{r_p} \cdot \overline{R}}{r_p R}$

Le déplacement radial atteint 40 cm au degré 2 et 3 mm au degré 3. Les déplacements horizontaux sont inférieurs au cm au degré 2 et au mm au degré 3.

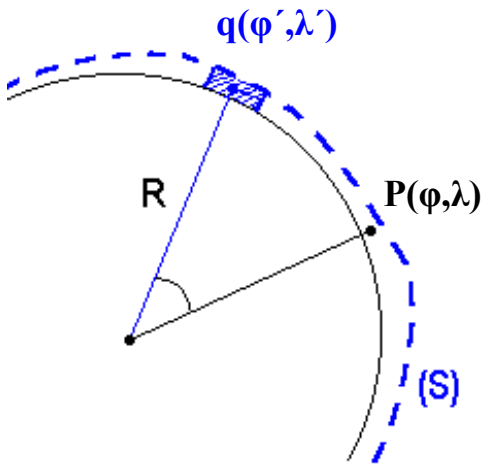
déformations de marée terrestre de degré 2  
en fonction de la latitude



déformations de marée terrestre de degré 2  
à l'équateur, en fonction de la longitude



## Potentiel et déformation de charge



Charge q de :

marée océanique :  $q = \xi_0 \rho_w$

pression atmosphérique :  $q = P_a / g$

rebond post-glaciaire :  $q = \xi_e \rho_c$

Le potentiel de simple couche généré par cette charge s'exprime en P à la surface (S) :

$$U_p(\varphi, \lambda) = 4\pi GR \sum_{l=1}^{\infty} \frac{1}{2l+1} \sum_{m=0}^l P_{lm}(\sin \varphi) (q_{lm}^c \cos m\lambda + q_{lm}^s \sin m\lambda) = 4\pi GR \sum_{l=1}^{\infty} \frac{q_l(\varphi, \lambda)}{2l+1}$$

soit en posant :  $g = \frac{GM}{R^2}$  et  $M = \frac{4}{3}\pi R^3 \rho_e$ , d'où  $4\pi gR = \frac{3g}{\rho_e}$  avec  $\rho_e = 5520 \text{ kg m}^{-3}$

$$U_p(\varphi, \lambda) = \frac{3g}{\rho_e} \sum_{l=1}^{\infty} \frac{1}{2l+1} \sum_{m=0}^l q_{lm}(\varphi, \lambda) = \sum_{l=1}^{\infty} U_l$$

D'après la première hypothèse de Love, le déplacement de la croûte visco-élastique est proportionnel au potentiel de charge :

$$\xi(\varphi, \lambda) = \sum_{l=1}^{\infty} h'_l \frac{U_l}{g} = \frac{3}{\rho_e} \sum_{l=1}^{\infty} \frac{h'_l}{2l+1} q_l(\varphi, \lambda)$$

D'après la seconde hypothèse de Love, cette déformation génère un potentiel additionnel de déformation de charge:

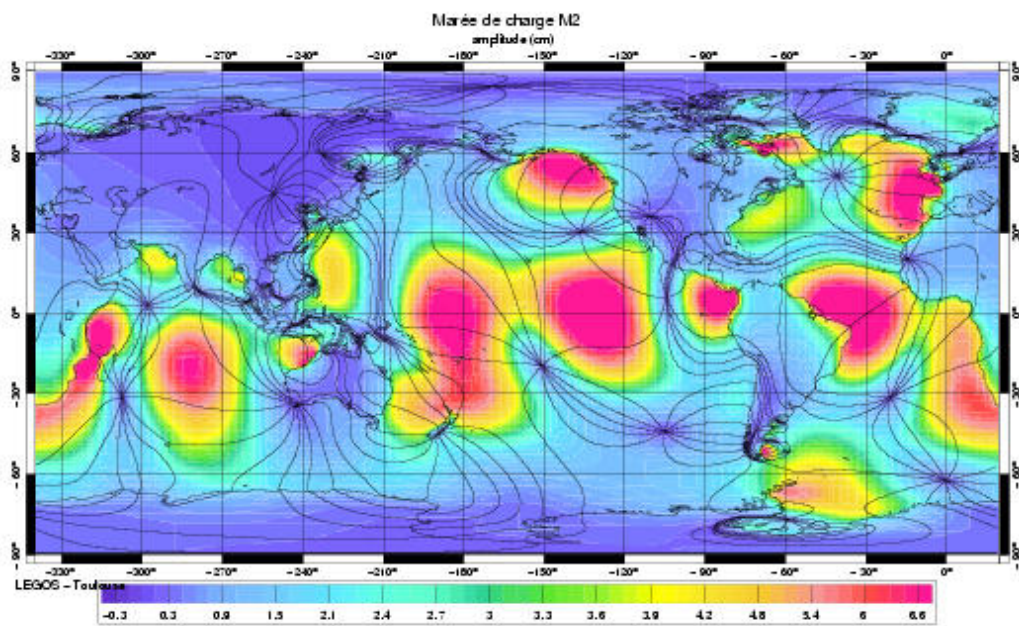
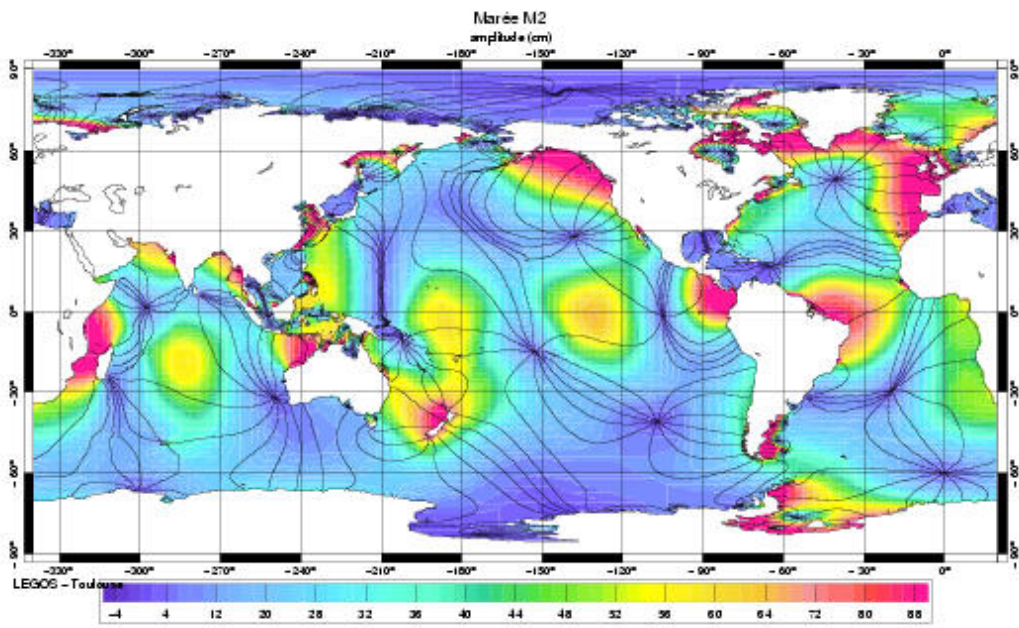
$$\Delta U_p(\varphi, \lambda) = \sum_{l=1}^{\infty} k'_l U_l = \frac{3g}{\rho_e} \sum_{l=1}^{\infty} \frac{k'_l}{2l+1} q_l(\varphi, \lambda)$$

Le potentiel total de charge :  $U_p + \Delta U_p$  peut s'écrire sous une forme similaire au potentiel de volume dont les coefficients de Stokes s'expriment alors :

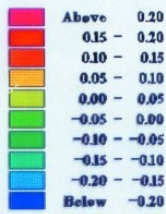
$$\begin{Bmatrix} \Delta C_{lm} \\ \Delta S_{lm} \end{Bmatrix} = \frac{4\pi R^2}{M} \frac{1+k'_l}{2l+1} \begin{Bmatrix} q_{lm}^c \\ q_{lm}^s \end{Bmatrix} = \frac{3}{R\rho_e} \frac{1+k'_l}{2l+1} \begin{Bmatrix} q_{lm}^c \\ q_{lm}^s \end{Bmatrix}$$



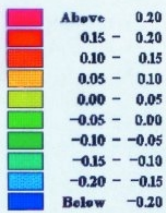
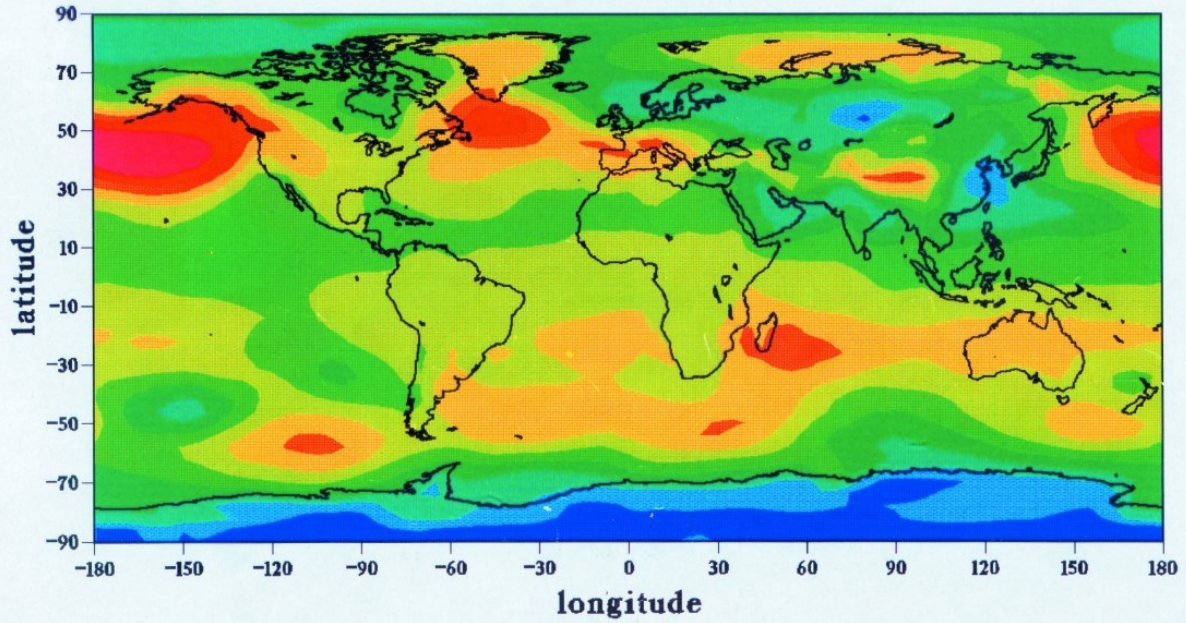
## Marée et effet de charge M2 (modèle FEZ-2002)



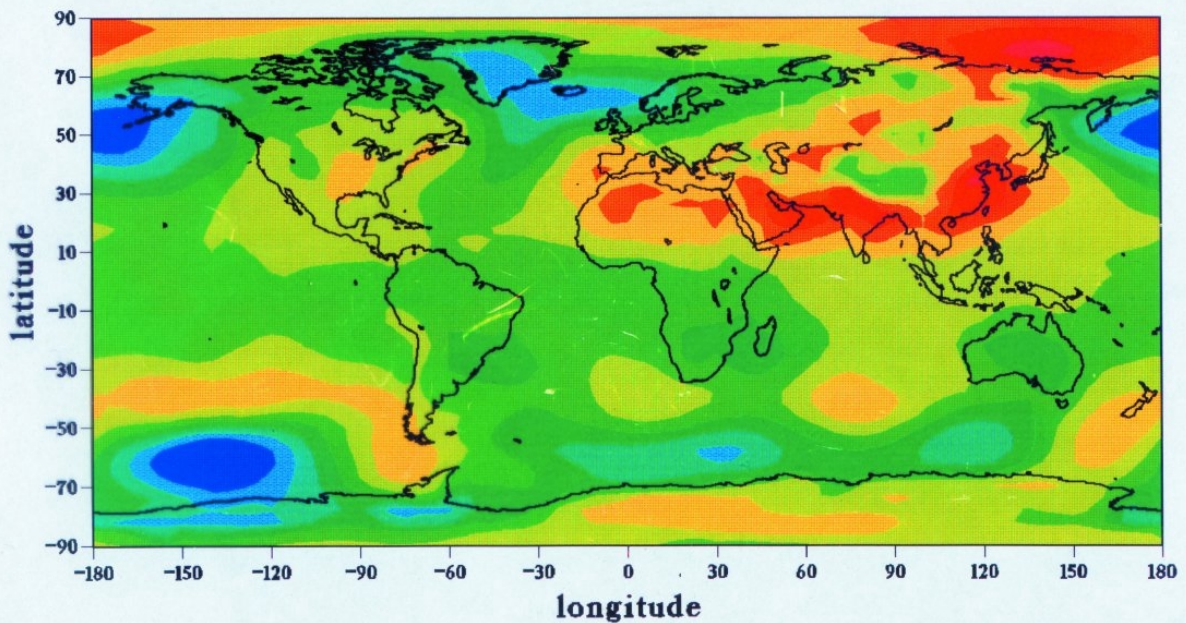




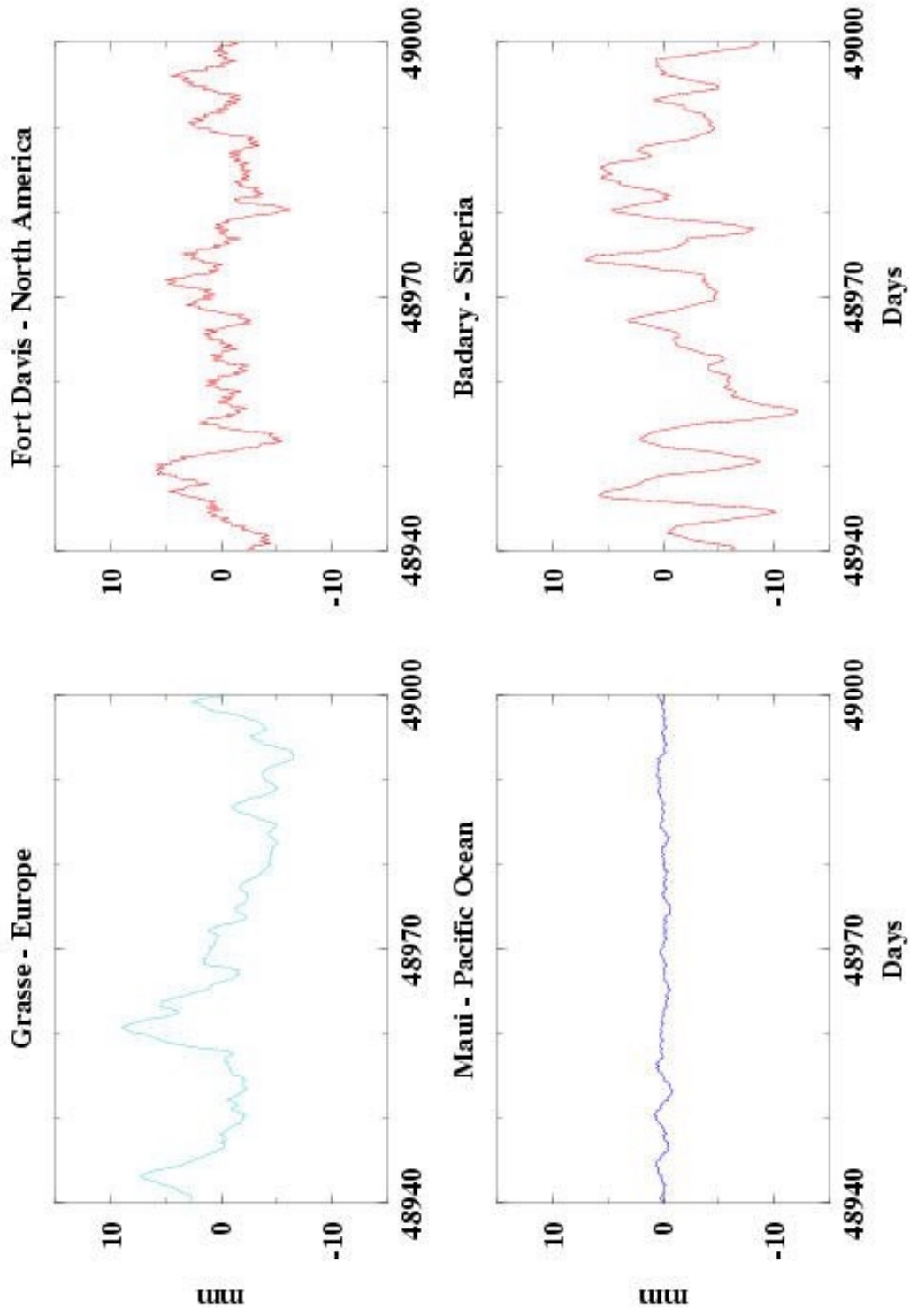
**MEAN ATMOSPHERIC PRESSURE VARIATIONS**  
 between FEBRUARY and AUGUST  
 converted in water height differences  
 (meters)



**MEAN ATMOSPHERIC PRESSURE VARIATIONS**  
 between JUNE and DECEMBER  
 converted to water height differences  
 (meters)

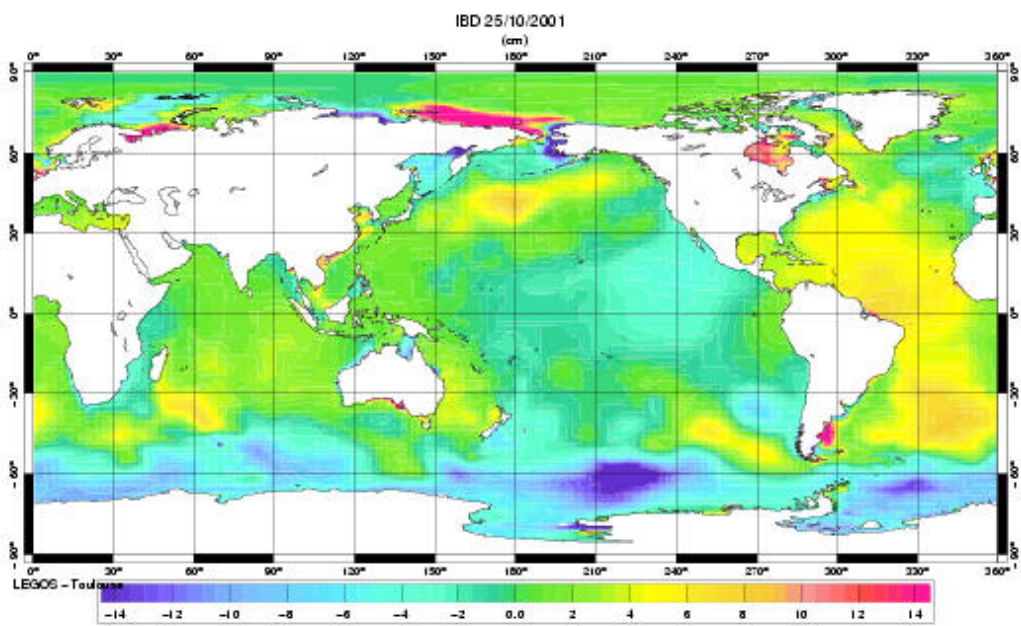
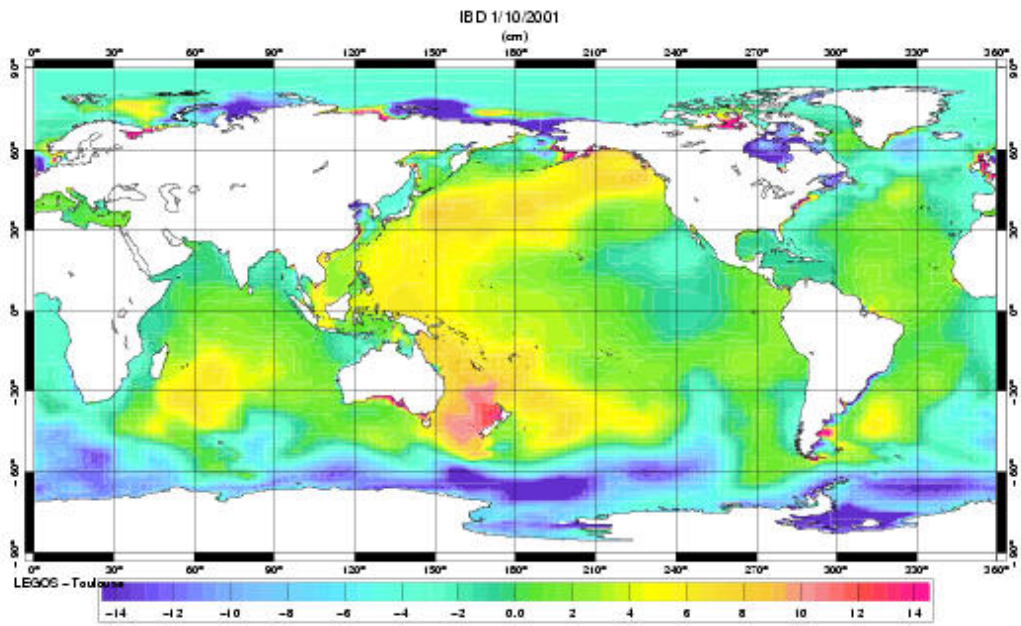


Vertical Displacement induced by global atmospheric and oceanic loading





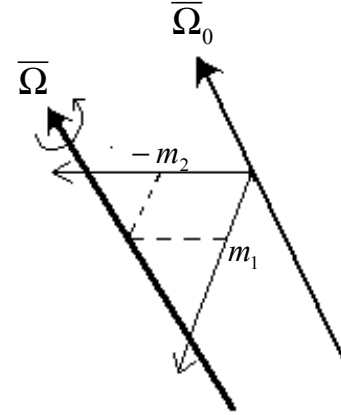
# Correction à l'effet baromètre inverse les 1<sup>er</sup> et 25 Octobre 2001



# Déformation de marée polaire

La variation du potentiel centrifuge à la surface de la Terre vaut :

$$\Delta V_c = -\frac{\Omega_o^2 R^2}{3} P_{21}(\sin \varphi)(m_1 \cos \lambda + m_2 \sin \lambda)$$



Le déplacement de la surface de la Terre induit par la variation du potentiel centrifuge représenté par l'harmonique sphérique de degré 2 s'exprime conformément à la formulation de Love :

$$u_r = \frac{h_2}{g} \Delta V_c = -\frac{h_2}{g} \Omega_o^2 R^2 \sin \varphi \cos \varphi (m_1 \cos \lambda + m_2 \sin \lambda)$$

$$u_\varphi = \frac{\ell_2}{g} \frac{\partial \Delta V_c}{\partial \varphi} = -\frac{\ell_2}{g} \Omega_o^2 R^2 \cos 2\varphi (m_1 \cos \lambda + m_2 \sin \lambda)$$

$$u_\lambda = \frac{\ell_2}{g \cos \varphi} \frac{\partial \Delta V_c}{\partial \lambda} = \frac{\ell_2}{g} \Omega_o^2 R^2 \sin \varphi (m_1 \sin \lambda - m_2 \cos \lambda)$$

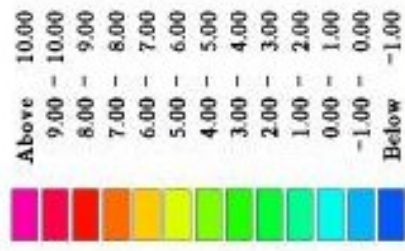
soit en remplaçant les nombres sans dimension  $m_1, m_2$  par les coordonnées du pôle  $\bar{x}_p = x_p - \bar{x}, -\bar{y}_p = -(y_p - \bar{y})$  exprimées en radian par rapport au pôle moyen de rotation  $(\bar{x}, \bar{y})$  dans le système classique d'axes polaires :

$$u_r = -13.2 \cdot 10^3 \sin \varphi \cos \varphi (\bar{x}_p \cos \lambda - \bar{y}_p \sin \lambda)$$

$$u_\varphi = -1.9 \cdot 10^3 \cos 2\varphi (\bar{x}_p \cos \lambda - \bar{y}_p \sin \lambda)$$

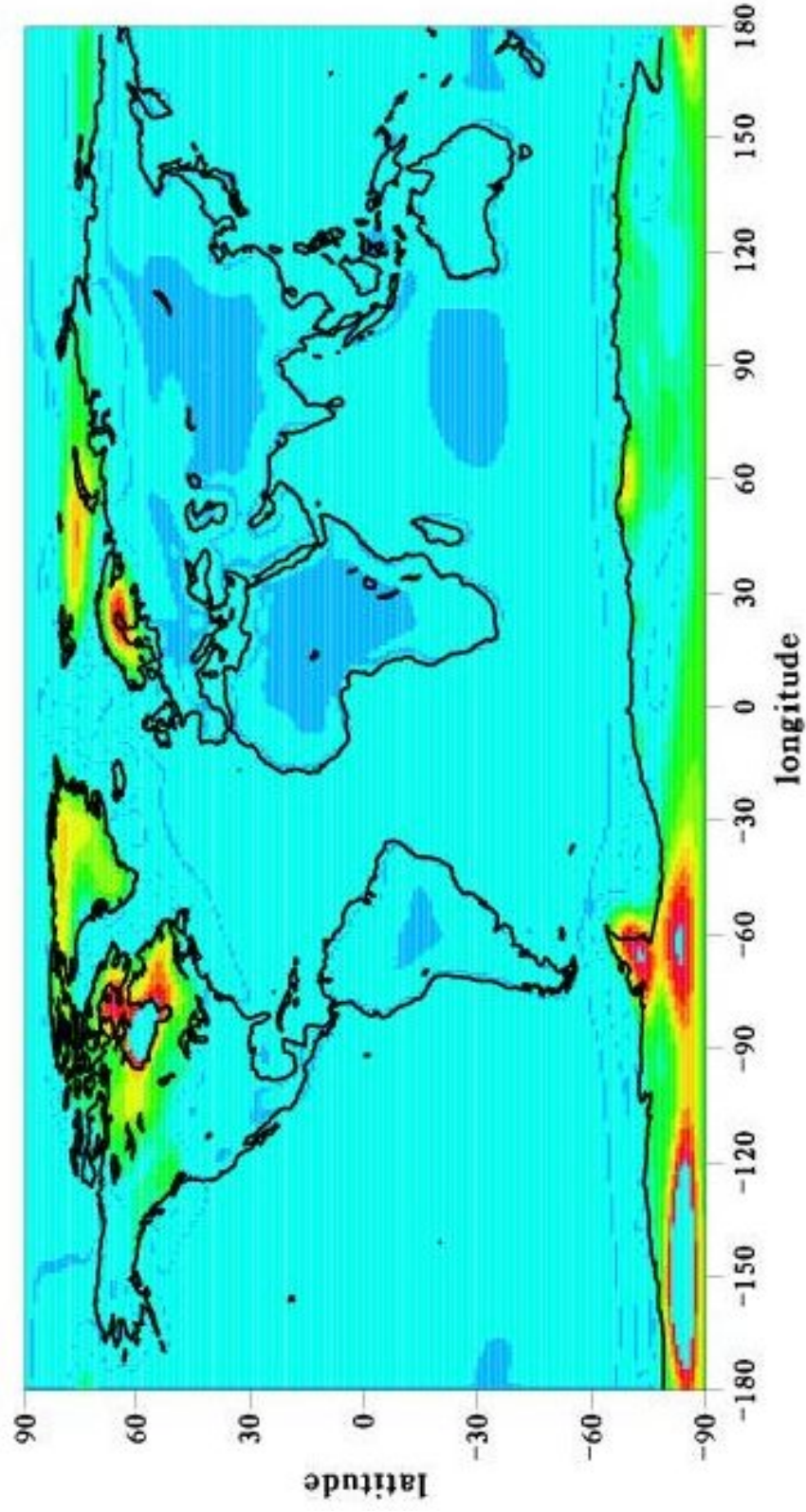
$$u_\lambda = -1.9 \cdot 10^3 \sin \varphi (\bar{x}_p \sin \lambda - \bar{y}_p \cos \lambda)$$

**Le déplacement radial peut atteindre 2 cm**



**modele de rebond post-glaciaire ICE-4G**  
(Peltier et al., 1995)  
**vitesse de la deformation en mm/an**

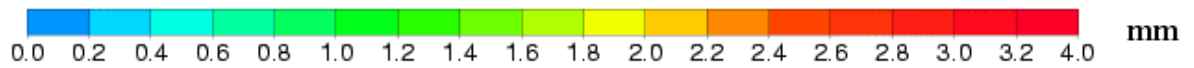
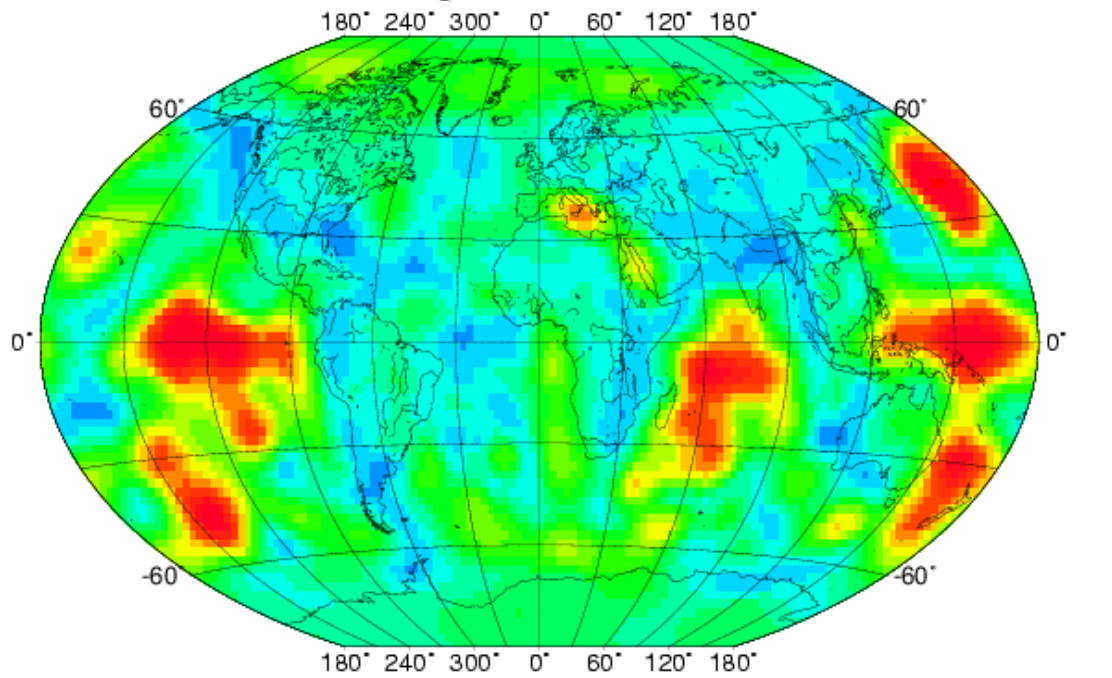
(rms : 0.91 / moy : 0.38 / min : -0.90 / max : 9.91)



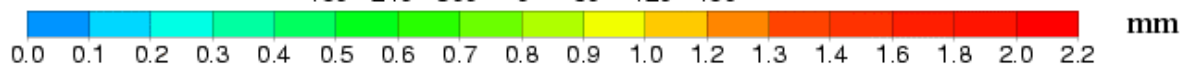
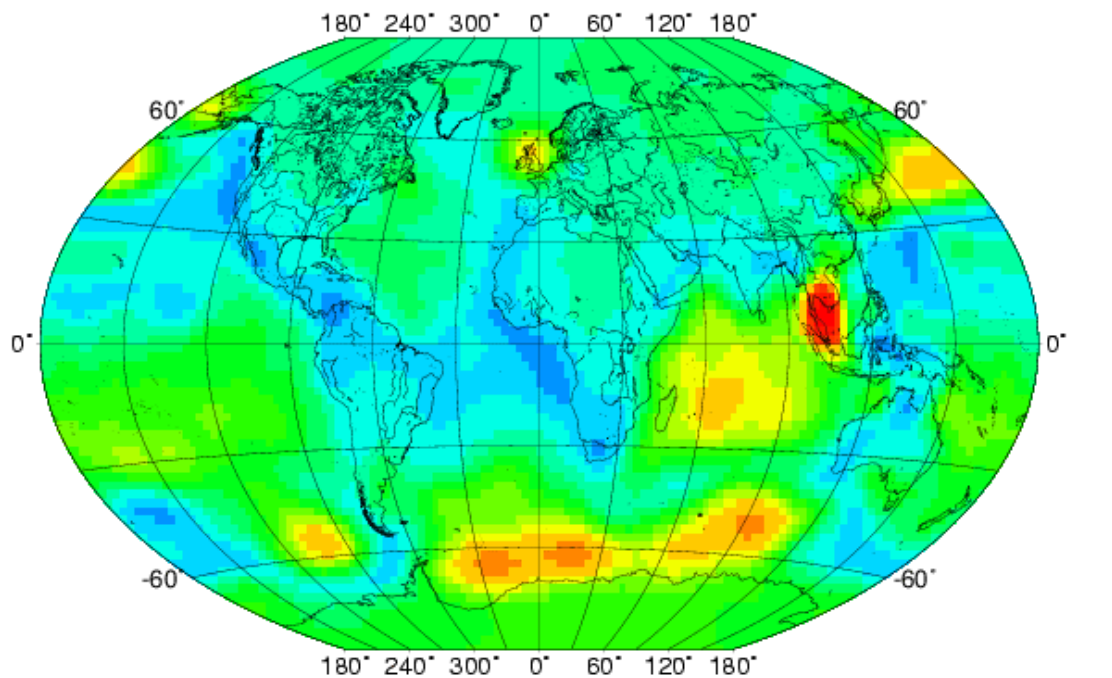
# VERTICAL CRUSTAL DEFORMATION (ocean)

- Annual amplitude (mm) -

- Topex/Poseidon data -



- POCM model -



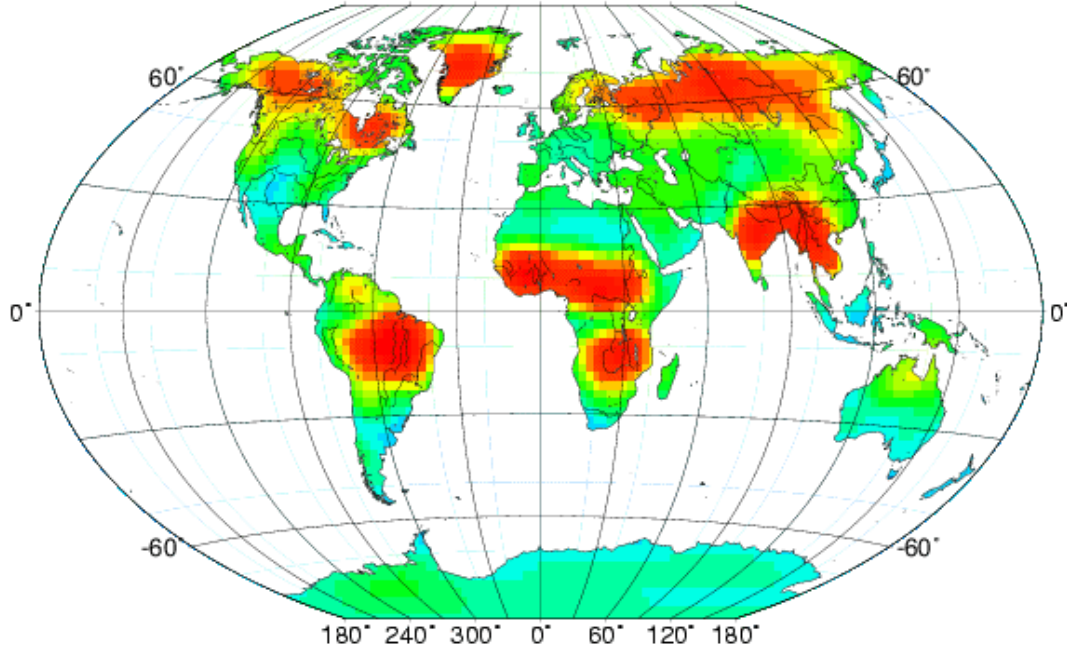


**VERTICAL CRUSTAL DEFORMATION caused by SOIL MOISTURE and SNOW**

**- Annual amplitude (mm) -**

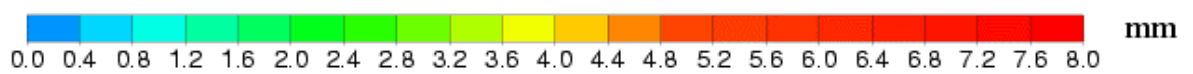
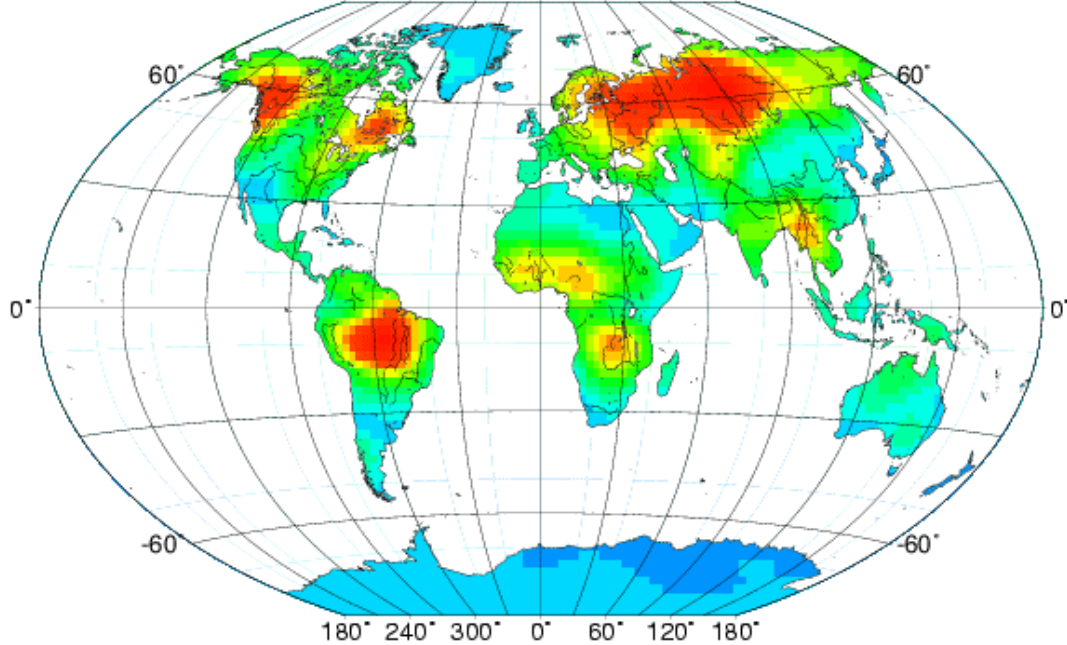
**- Huang et al. (1996) and ISLSCP-**

180° 240° 300° 0° 60° 120° 180°



**- GSWP data -**

180° 240° 300° 0° 60° 120° 180°

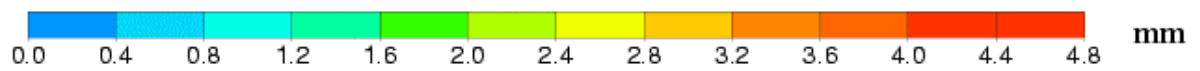
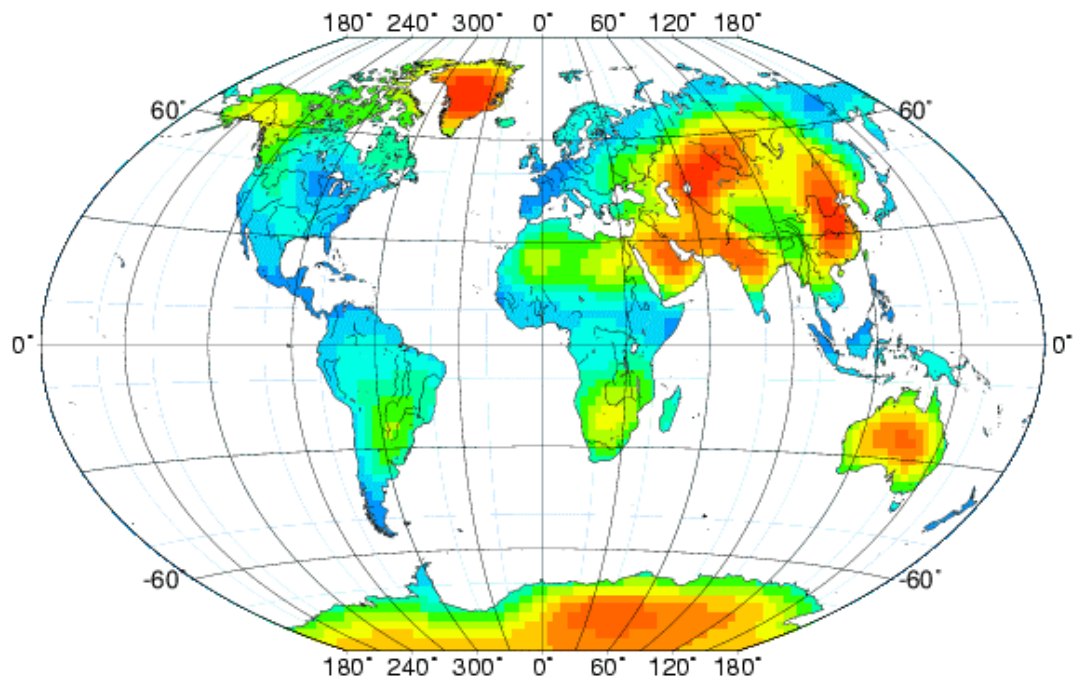




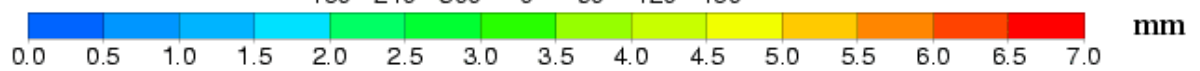
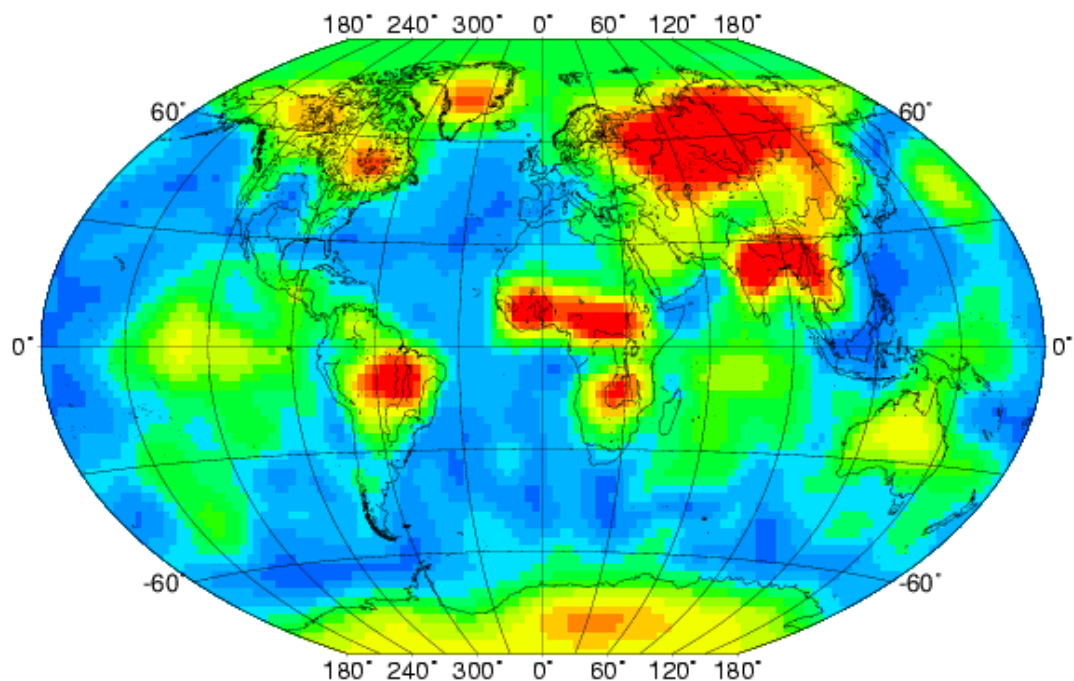
## VERTICAL CRUSTAL DEFORMATION

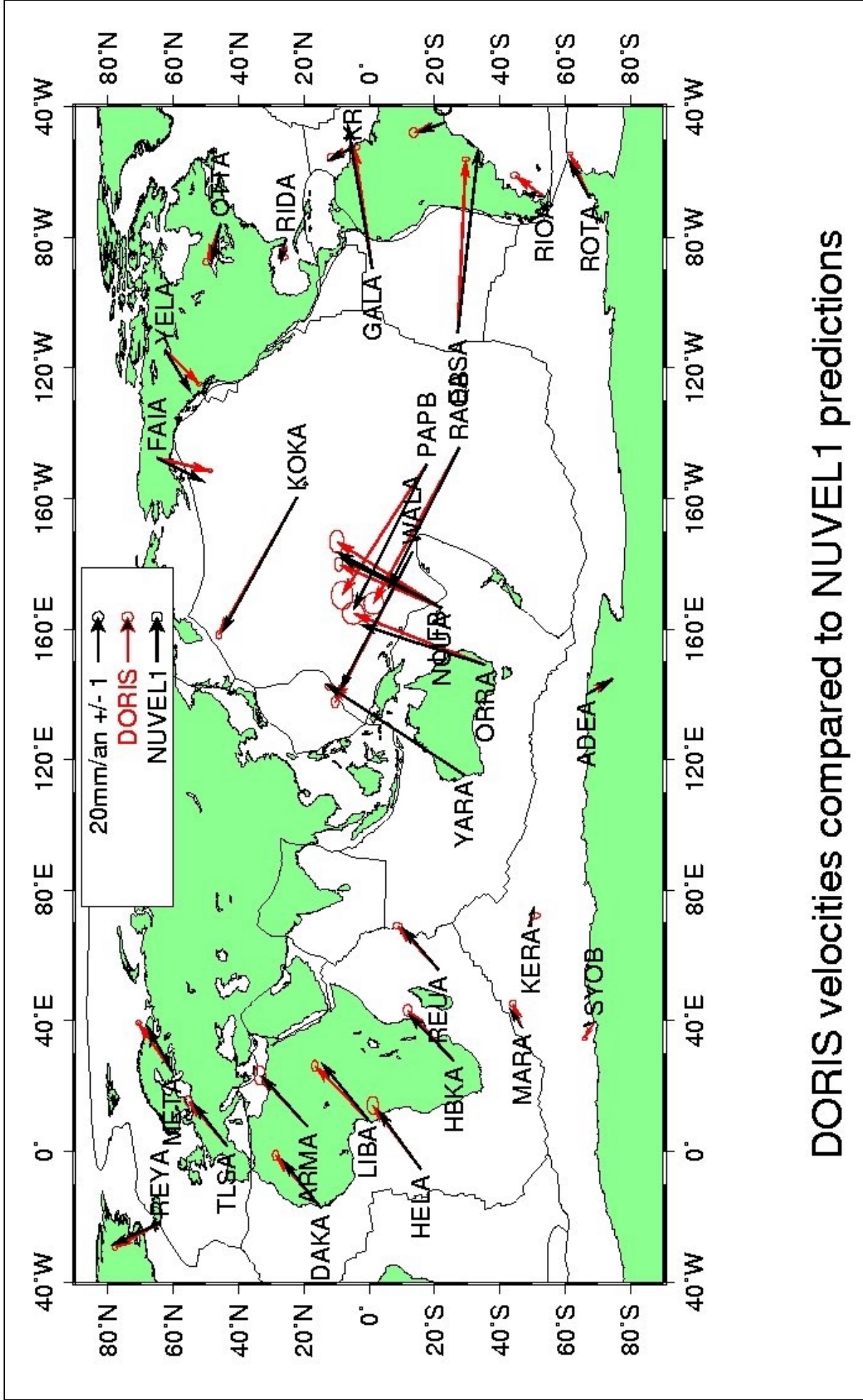
- Annual amplitude (mm) -

- PRESSURE contribution NCEP data -



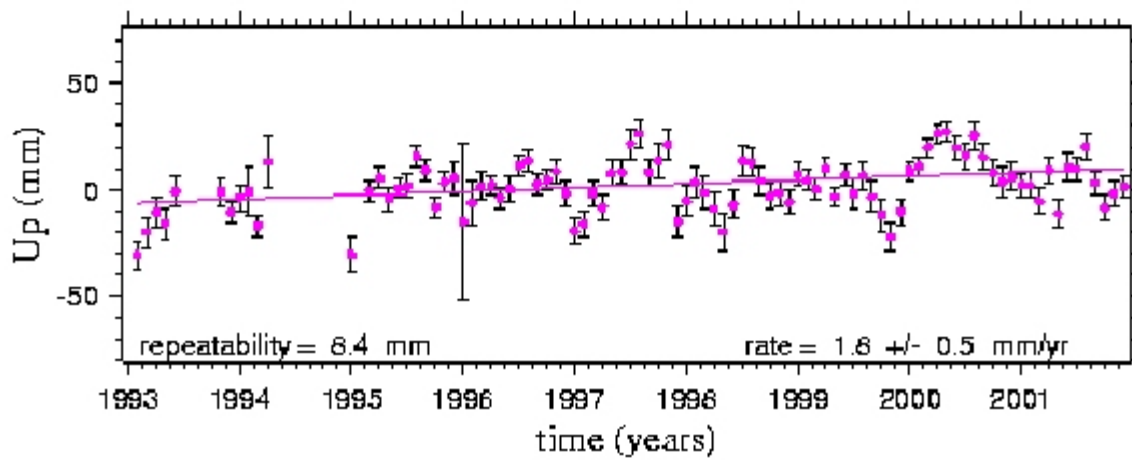
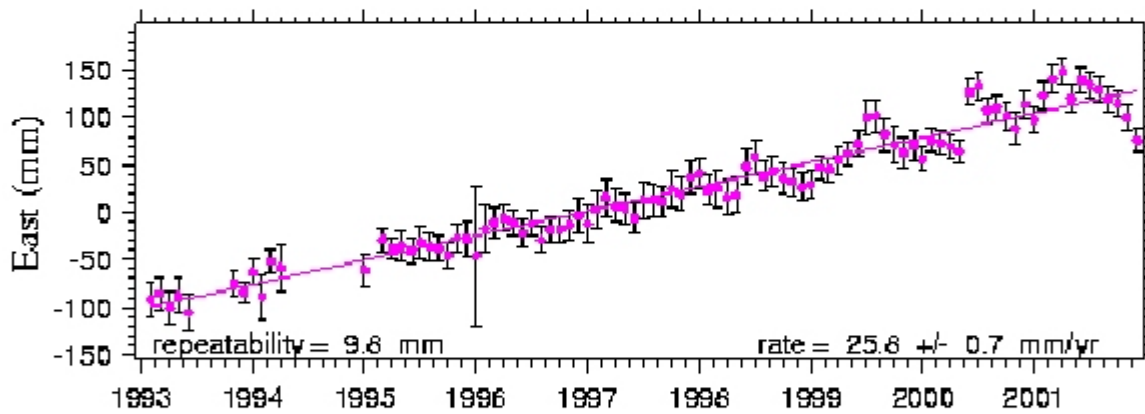
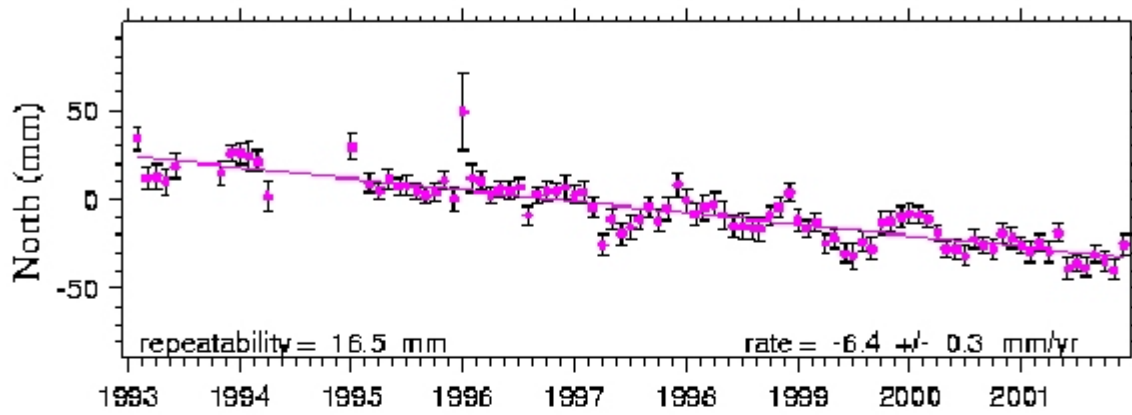
- TOTAL CONTRIBUTION (Huang et al., ISLSCP, Topex/Poseidon, NCEP) -





DORIS velocities compared to NUVEL1 predictions

# BADA



# AREA

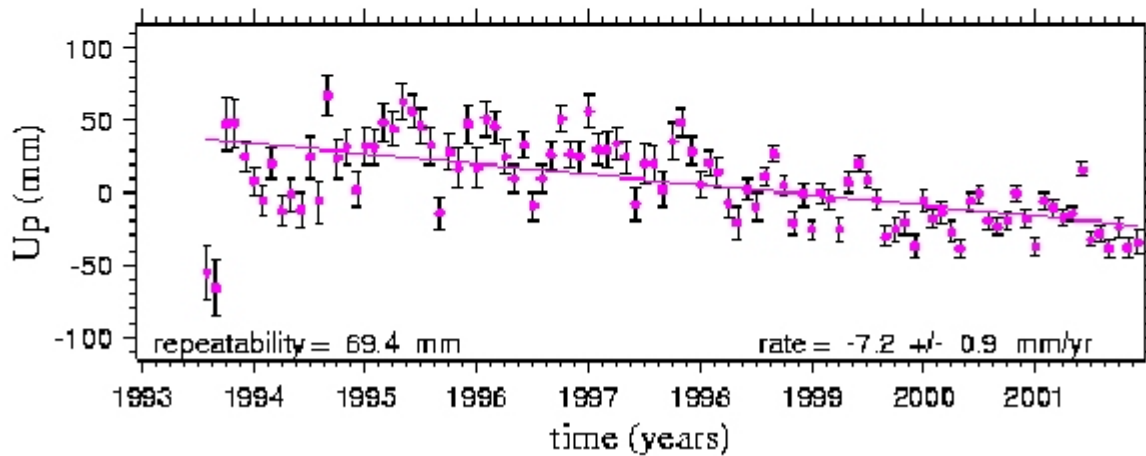
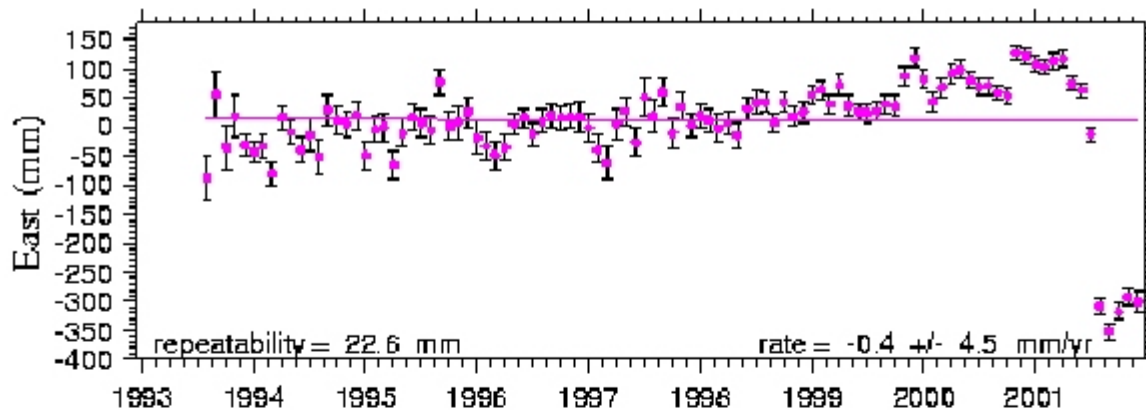
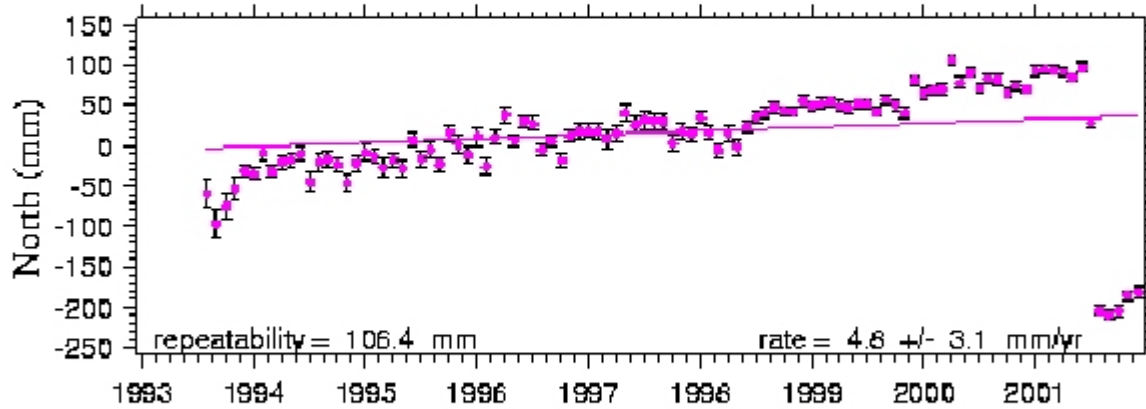


Figure 4

### Geocenter variations

Climatology (solid), Geodesy (dashed)

